1. History

(a) What is the sieve of Eratosthenes?
(b) Is it true that there exists a consecutive sequence of 521 integers, all of which are non-prime?
(c) What is an Euler circuit?
(d) In Euler’s representation of the layout of Konigsberg as a graph, what physical entity is represented by an edge and what physical entity is represented by a vertex?

2. Equivalence Relation. On the set $N \times N$, define the relation $\sim$ by $(a, b) \sim (c, d)$ if $a + b = c + d$. State whether or not this is an equivalence relation. If your answer is “no”, prove it. If your answer is “yes”, prove it and sketch some of the equivalence classes on a 2-d plane.

3. Relations. Let $R_1$ and $R_2$ be binary relations on set $S$. For each of the following, either prove the expression is always true, or give a counterexample demonstrating that it can be false.

(a) If $R_1$ and $R_2$ are symmetric, then $R_1 \cap R_2$ is symmetric.
(b) If $R_1$ is antisymmetric, then the relation $R_2$ defined by $\{(x, y) : x, y \in S, (x, y) \notin R_1\}$ is antisymmetric.

4. More Relations.

Fill in the following table, indicating which properties hold for each of the three binary relations on the set $S = \{1, 2, 3\}$. (Fill in all boxes, putting a check mark if the property holds, and an “X” if it does not.)

<table>
<thead>
<tr>
<th>Relations</th>
<th>Reflexive</th>
<th>Antireflexive</th>
<th>Symmetric</th>
<th>Antisymmetric</th>
<th>Transitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>${(1, 2), (2, 1), (2, 3)}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>${(1, 2), (2, 1), (1, 1), (2, 2)}$</td>
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</tbody>
</table>

5. Quantification. For each of the following expressions, state whether it is true or not. Give an informal argument (this does not have to be a rigorous proof) about why your answer is correct. (Although the argument does not have to be a rigorous proof, it must be a convincing and accurate argument supporting your answer.)

(a) $\forall x \in N \exists y \in N [x > 0 \rightarrow x + y \text{ even}]$
(b) $\exists y \in N \forall x \in N [x > 0 \rightarrow x + y \text{ even}]$
(c) \( \exists x \in \mathbb{N} \forall y \in \mathbb{N} \ [x > 0 \rightarrow x + y \text{ even}] \)

6. \textbf{Big-Oh}. For \( f, g : \mathbb{N} \rightarrow \mathbb{N} \) defined by \( f(n) = 100n \) and \( g(n) = n^2 \) do the following:

(a) Prove that \( f(n) \) is \( O(g(n)) \).
(b) Prove that \( g(n) \) is not \( O(f(n)) \).