Constructing Proofs

Problem 1: Feedback (10 points)
We want to use the first problem of each homework to get your feedback. You have to answer these questions and they will be graded. For this week, the questions are:
(a) What year are you in?
(b) What other classes are you taking this semester? Please give the Course Number and the Course Name.
(c) How do you find the lectures? Do you have any suggestion for making the lectures better suited to your interests?
(d) How did you find this first homework? Please answer after you finish the homework.

Problem 2: Direct Proof/Modus Ponens (30 points)
(a) Prove that, for all integers $n$, if $n$ is odd then $n^2$ is odd.
(b) Prove that, for all integers $n$, $n^3 - n$ is divisible by 3.
(c) Prove that, for all real numbers $x$, $2x^2 - 4x + 3 > 0$.

Problem 3: Proofs/Counter-Examples/Quantifiers (30 points)
(a) Is the following proposition true or false: "For every prime $p$, $p+2$ is prime". If true, give a proof. If false, give a counter-example.
(b) Is the following proposition true or false: "There exist three consecutive odd integers that are primes, that is, for some odd number $p$, $p$, $p+2$, and $p+4$ are primes". If true, give a proof. If false, give a counter-example.
(c) Is the following proposition true or false: "For every pair of (not necessarily distinct) irrational numbers $x$ and $y$, the product $x \cdot y$ is irrational". If true, give a proof. If false, give a counter-example.

Problem 4: Puzzle (Modus Ponens) (30 points)
(a) Prove that an $8 \times 8$ chessboard can be completely covered using dominoes (2 $\times$ 1 pieces).
(b) Prove that it is impossible to cover completely with dominoes the $8 \times 8$ chessboard with one square at a corner of the board removed.
(c) Prove that it is impossible to cover completely with dominoes the $8 \times 8$ chessboard with two squares at opposite corners of the board removed.