Problem 1: Feedback (10 points)
Share a joke to entertain the class! Can you come up with a joke that includes the words "induction", "recursion" and/or "algorithm"?

Problem 2: Solving Recurrences (60 points)
Solve the following recurrences. Please show all your work, including the work to guess the form of the solution, and the work to prove the solution that you claim.
(a) $a_n = 2a_{n-1} + 1$, $a_1 = 1$. (Notice that this recurrence is like the Towers of Hanoi problem).
(b) $a_n = 2a_{n-1} + 7$, $a_1 = 1$.
(c) $a_n = 3a_{n-1} + 7$, $a_1 = 1$.
(d) $a_n = a_{n/2} + 1$, $a_1 = 0$, and $n$ is a power of 2, say $n = 2^k$. (Notice that this recurrence is like the "find the heaviest ball" problem).
(e) $a_n = a_{n/2} + 7$, $a_1 = 0$, and $n$ is a power of 2, say $n = 2^k$.
(f) $a_n = a_{n/3} + 7$, $a_1 = 0$, and $n$ is a power of 3, say $n = 3^k$.

Problem 3: Algorithm Design (30 points)
In class we showed how to find the maximum of $n$ distinct elements using $n - 1$ comparisons. Similarly, we can find the minimum of $n$ distinct elements using $n - 1$ comparisons. Therefore, we can find both the maximum and the minimum of $n$ distinct elements using $2(n-1) = 2n-2$ comparisons. Can we do better?

Can you find the maximum and minimum of $n = 4$ distinct elements using only 4 comparisons (notice that $2(n-1) = 2(4-1) = 6$)? Let us assume that $n$ is even, or even that $n$ is a power of 2. Can you find the maximum and minimum of $n$ elements using only $3 \frac{n}{2} - 2$ comparisons?