Constructing Proofs

Problem 1: Feedback (10 points)
What did you think about Strassen’s multiplication algorithm?

Problem 2: (10 points)
Write the definition of the $O()$ notation. Prove that:
(a) $n + \sqrt{n} + 5 = O(n)$,
(b) $n^2 + n + \log n = O(n^2)$.

Problem 3: (20 points)
Recall the definition of the log function: $x^z = y$ if and only if $\log_x z = y$.
Recall also that $a^{\log_x a} = x$.
(a) Prove that $\log_a b = \frac{\log_e b}{\log_e a}$.
(b) Using the definition of the $O()$ notation, prove that $n \log_2 n = O(n \log_e n)$ and that $\log^2 n = O(\log^2 e n)$.

Problem 3: (10 points)
Argue that:
(a) $p^2 \log_2 n = n^{1.5}$.
(b) $4^{\log_2 n} = n^2$.
(c) $2^{\log_2^2 n} = n^{\log_2 n}$.

Problem 4: (40 points)
Rank the following functions by order of growth, that is, give an arrangement $g_1, g_2, \ldots$, such that $g_1 = O(g_2)$, $g_2 = O(g_3)$, and so on. Give proofs.
$\sqrt{2^{\log_2 n}}, n^3, n \log_2 n, \log_2^2 n, 2^{\log_2 n}, 2^{\log_2^2 n}, 4^{\log_2 n}$.

Problem 5: (10 points)
(a) Using induction on $n$, prove that, for all positive integers $n$, $n < 2^n$.
(b) Using the definition of $O()$ notation, prove that $n = O(2^n)$.
(c) Using the definition of $O()$ notation, prove that $2^n = O(2^{2n})$. 