Constructing Proofs

Problem 1: Feedback (10 points)
What did you think about Primality Testing?

Problem 2: (10 points)
Prove that:
(a) $5n^2 + 3n^3 - 25n + \log n = O(n^3)$. (b) $n^3 = O(2^n)$.

Problem 3: (20 points)
Prove that, for some integer $n_0$, $n^2 \leq 2^n$, for all $n \geq n_0$. You should explicitly compute $n_0$.

Problem 4: (30 points)
Rank the following functions by order of growth, that is, give an arrangement $g_1, g_2, \ldots$, such that $g_1 = O(g_2)$, $g_2 = O(g_3)$, and so on. Give proofs.
$2^n, 3^n, 2\sqrt{n}, n2^n, 2^{\log_2 n}, 2^{3\log_2 n}, 2^{3\log_2 n}, 3^{2\log_2 n}, n^n, n\sqrt{n}$.

Problem 5: (30 points)
For all integers $n$ between 2 and 11, and for all integers $a$, such that $1 \leq a \leq n - 1$, compute $a^{n-1} \mod n$. Report your observations.