Some Recent Progress in Lattice-Based Cryptography

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TCC 2009

Lattice-Based Cryptography

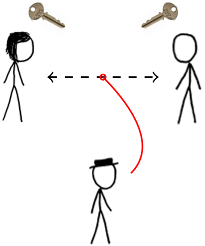
$y = g^x \bmod p$

$m^e \bmod N$

$N = p \cdot q$

$e(g^a, g^b)$

$\Longrightarrow$



(Images courtesy xkcd.org)

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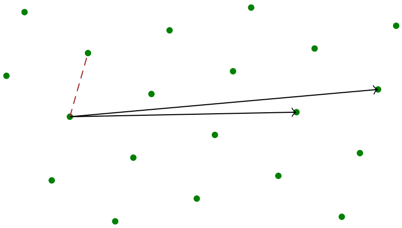

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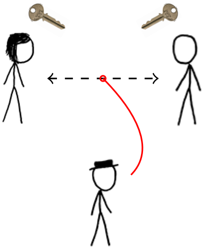

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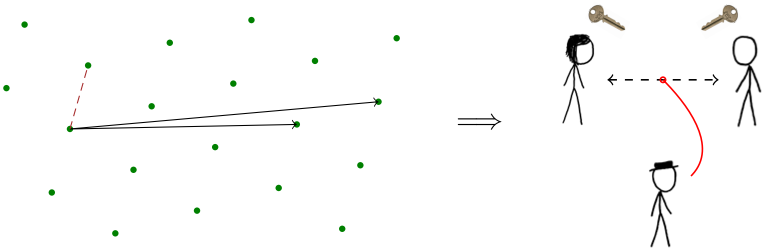

Why?

- Simple & efficient: linear, parallelizable
- Resists subexp & quantum attacks (so far)
- Security from worst-case assumptions [Ajtai96, . . .]

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If We Had 6 Hours. . .

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- Worst-case / average-case reductions

  [Aj96,AD97,CN97,Mi03,Re03,MR04,Re05,Pe07,GPV08,Pe09,. . .]

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- Cryptanalysis & concrete parameters

  [LLL82,Sc87,BKW00,AKS01,NR06,GN08,NV08,MR08,...]

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  - ⋆ *Efficiency* — complements general techniques
  - !! *Functionality* — uses 'extra features' of ideals

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  - ⋆ *Efficiency* — complements general techniques
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- Complexity of lattice problems
  - ⋆ Hardness [vEB81,Aj98,CN99,Mi00,Kh05,RR06,HR07,...]
  - ⋆ Limits on hardness [LLS90,Ba93,GG97,Ca98,AR04,GMR05,LLM06,P07,...]

This Talk

Hard Avg-Case Problems

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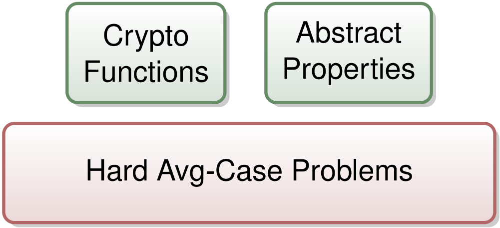
Crypto Functions
Abstract Properties
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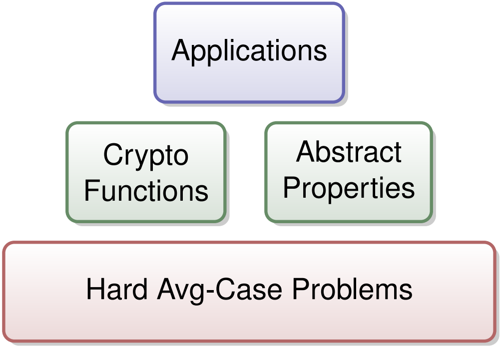
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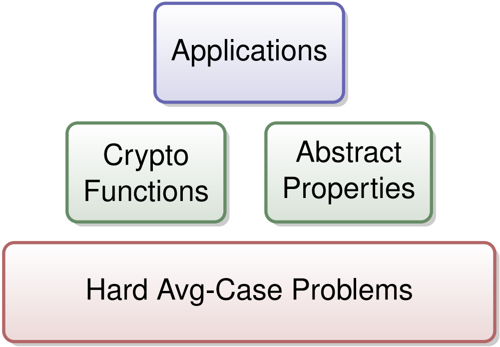

Goals

1. ‘De-mystify’ lattice-based crypto

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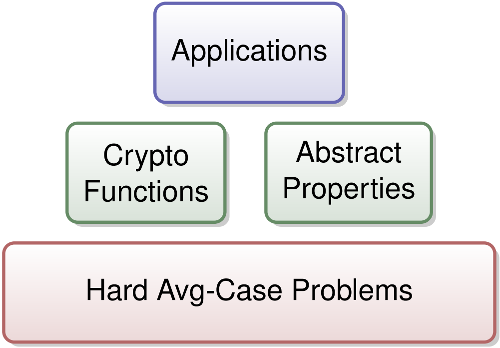

Goals

1. 'De-mystify' lattice-based crypto
2. Advocate a geometric perspective

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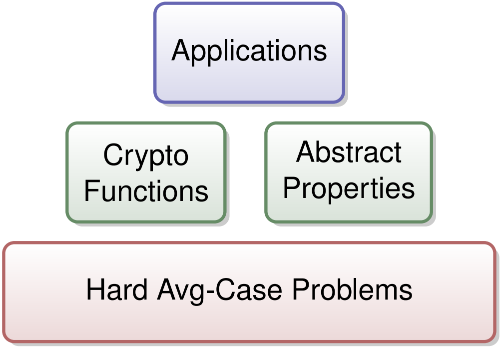
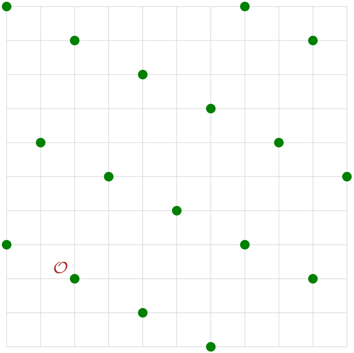


Goals

1. 'De-mystify' lattice-based crypto
2. Advocate a geometric perspective
3. Answer your questions

Lattices

- Today: full-rank subgroup $\mathcal{L}$ of $\mathbb{Z}^m$ $(\mathbf{x}, \mathbf{y} \in \mathcal{L} \Rightarrow \mathbf{x} \pm \mathbf{y} \in \mathcal{L};\ \dim \operatorname{span} = m)$



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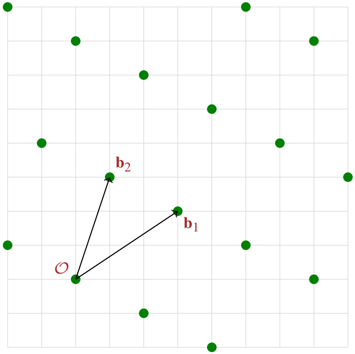
$$\mathcal{L} \quad = \quad \sum_{i=1}^{m} (\mathbb{Z} \cdot \mathbf{b}_i)$$



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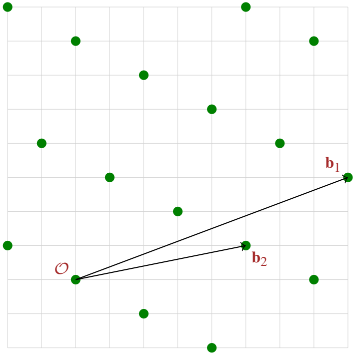

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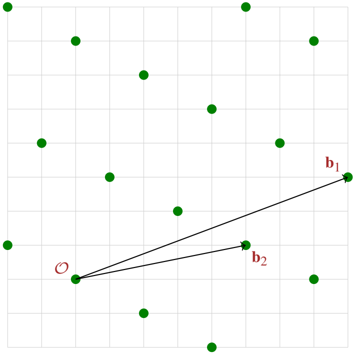

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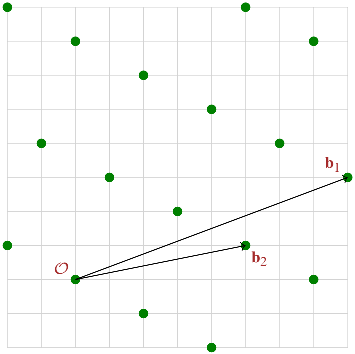

Hard Computational Problems

- Find ‘relatively short’ (nonzero) vectors
- Estimate geometric quantities (minimum distance, covering radius, . . .)

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$$z_1 \cdot \begin{pmatrix} | \\ \mathbf{a}_1 \\ | \end{pmatrix} + z_2 \cdot \begin{pmatrix} | \\ \mathbf{a}_2 \\ | \end{pmatrix} + \cdots + z_m \cdot \begin{pmatrix} | \\ \mathbf{a}_m \\ | \end{pmatrix} = \begin{pmatrix} | \\ 0 \\ | \end{pmatrix} \in \mathbb{Z}_q^n$$

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Hash Function [Ajtai96,GGH97]

- Set $m > n \lg q$. Define $f_{\mathbf{A}} : \{0,1\}^m \to \mathbb{Z}_q^n$

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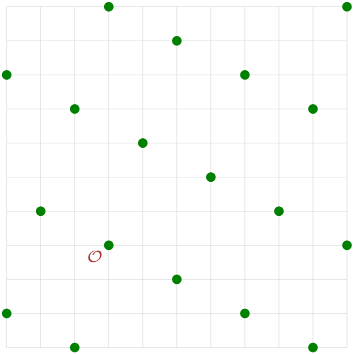
...yields solution $\mathbf{z} = \mathbf{x} - \mathbf{x}' \in \{0, \pm 1\}^m$.

Geometric Perspective

- 'Parity check' matrix $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$

$$\mathcal{L}^{\perp}(\mathbf{A}) = \{\mathbf{z} \in \mathbb{Z}^m \ : \ \mathbf{A}\mathbf{z} = \mathbf{0}\}$$



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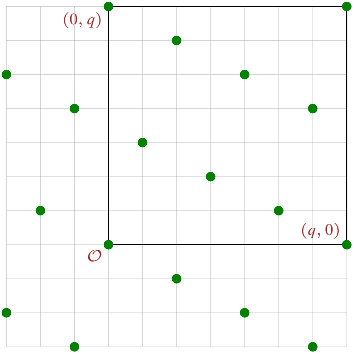

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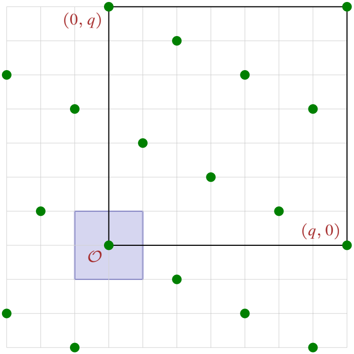

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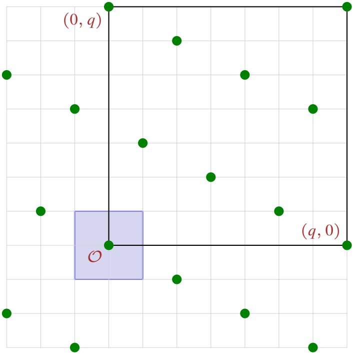

Average / Worst-Case Connection **[Ajtai96,...]**

Finding 'short' nonzero $\mathbf{z} \in \mathcal{L}^{\perp}(\mathbf{A})$

$\Downarrow$

approx lattice problems in worst case

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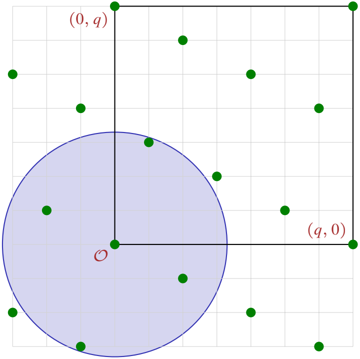

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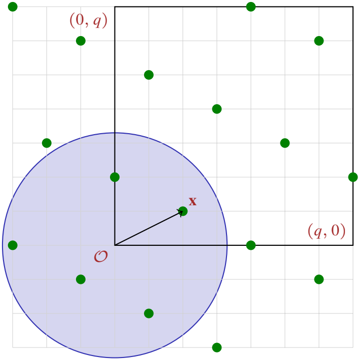

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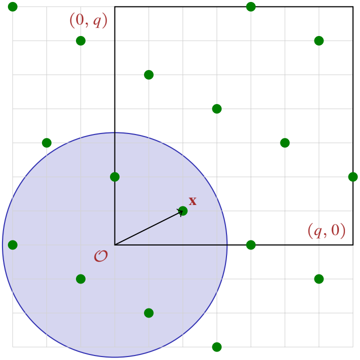
- Enlarge domain of $f_{\mathbf{A}}$ to  ...

...still O-W & C-R!



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Gaussians and Lattices

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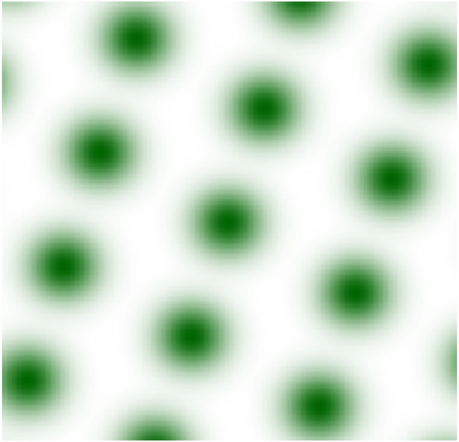


Gaussians and Lattices

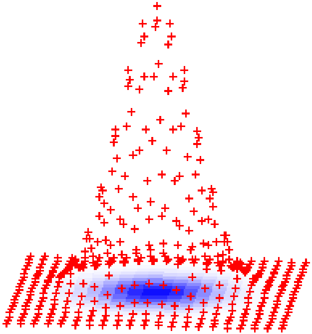
Gaussians and Lattices

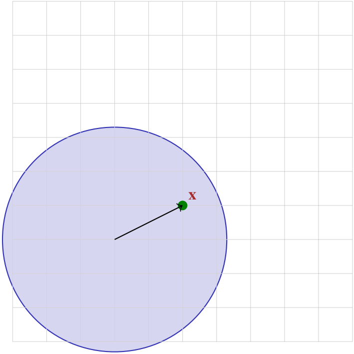
"Uniform" over $\mathbb{R}^m$ when std dev $\geq$ min basis length

(Used in worst/average-case reductions [Re03,MR04,. . .])

Discrete Gaussians

- Fix uniform $\mathbf{A}$. Choose Gaussian input $\mathbf{x} \in \mathbb{Z}^m$:





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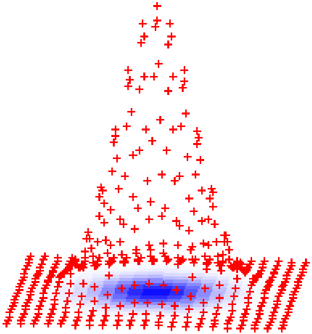

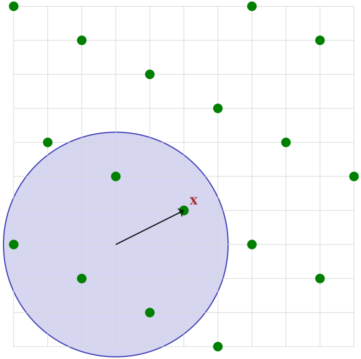

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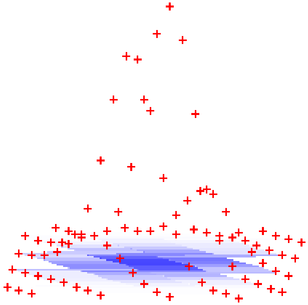

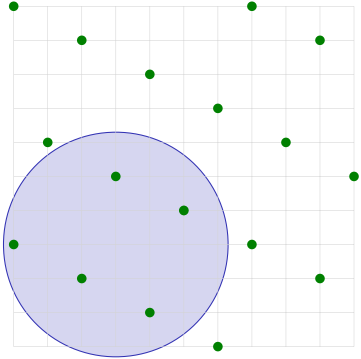

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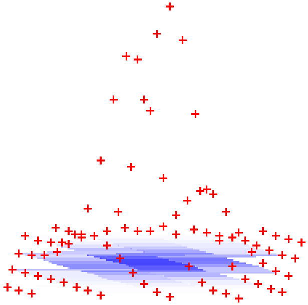
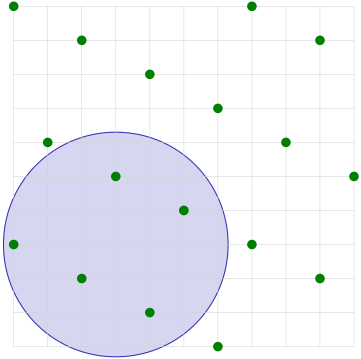
  (Analyzed in [Ba93,Re03,AR04,MR04,Re05,PR06,LM06,Pe07,...])





A ‘Master’ Trapdoor

Suitable ‘trapdoor’

$\Downarrow$

Invert $f_{\mathbf{A}}$ in a very strong sense

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Short basis $\mathbf{B}$ of $\mathcal{L}^{\perp}(\mathbf{A})$

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Given *any* short $\mathbf{B}$ and $\mathbf{u}$,

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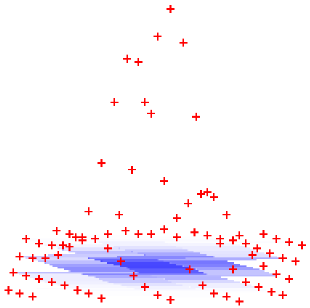

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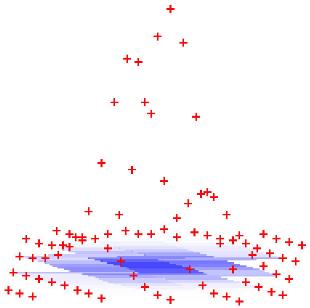

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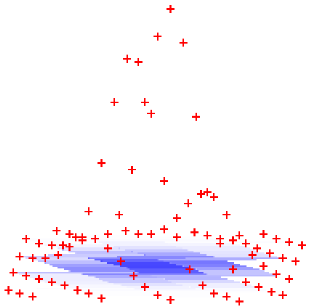
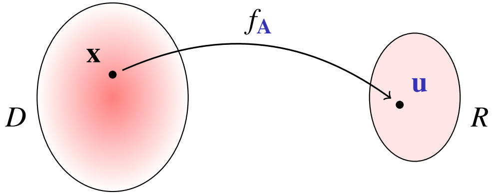
- Generate $\mathbf{A}$ with $\mathbf{B}$ [Aj99,AP09]



Abstractly: Preimage Sampleable Function



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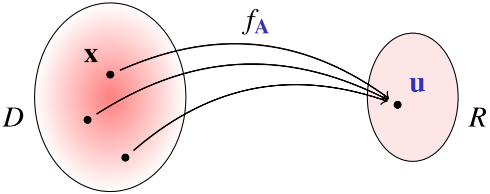

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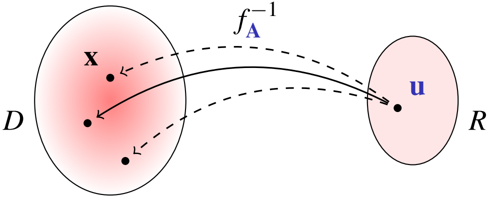

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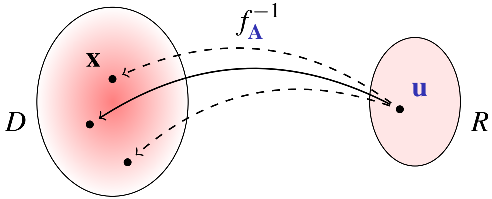

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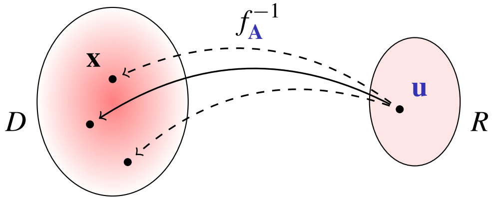

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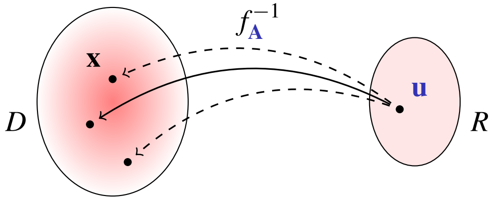

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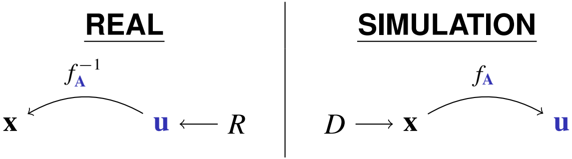

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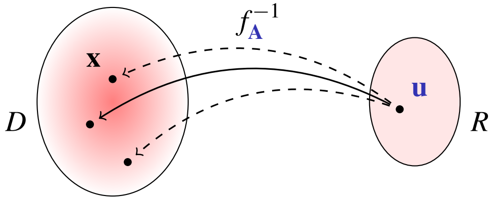

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REAL	**SIMULATION**
$\mathbf{x} \xleftarrow{f_{\mathbf{A}}^{-1}} \mathbf{u} \longleftarrow R$	$D \longrightarrow \mathbf{x} \xrightarrow{f_{\mathbf{A}}} \mathbf{u}$

- Apps: 'hash-and-sign' sigs [GPV08], NISZK [PV08], ...

Onward, to Cryptomania . . .

Learning with Errors

- Goal: distinguish 'noisy inner products' from uniform.

$$
\begin{array}{ccc}
\mathbf{a}_1 & , & b_1 = \langle \mathbf{a}_1 , \mathbf{s} \rangle + e_1 \\
\mathbf{a}_2 & , & b_2 = \langle \mathbf{a}_2 , \mathbf{s} \rangle + e_2 \\
 & \vdots &
\end{array}
$$

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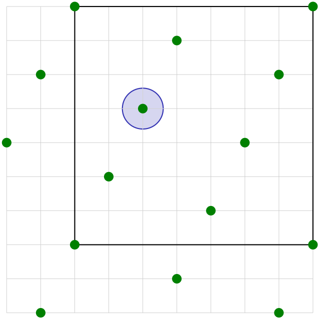
$$\mathcal{L}(\mathbf{A}) = \{\mathbf{z} \in \mathbb{Z}^m \; : \; \exists \mathbf{s}. \; \mathbf{z} \equiv \mathbf{A}^t\mathbf{s} \bmod q\}$$

'Bounded-distance' (unique) decoding



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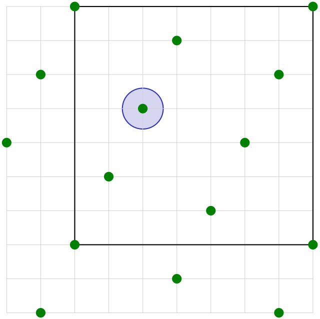

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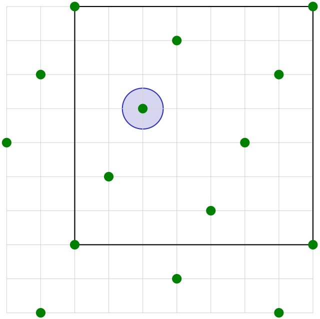
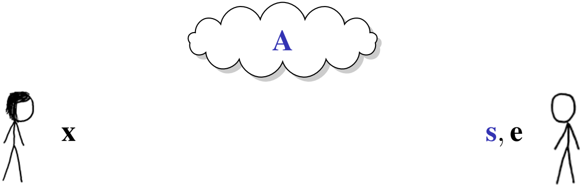


- Worst-case hardness [Re05,Pe09]
- Basis of much crypto

[Re05,PW08,GPV08,PVW08,CDMW08,AGV09,CPS09,. . .]

Key Agreement & Encryption



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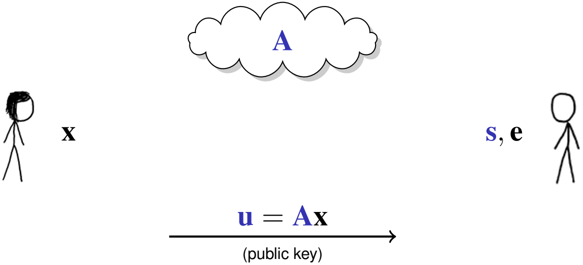

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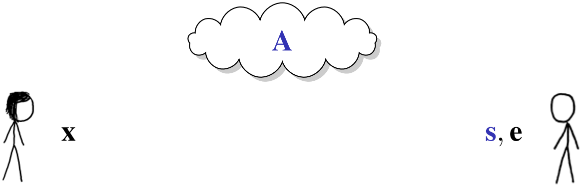

$$\mathbf{u} = \mathbf{A}\mathbf{x}$$

$\longrightarrow$

(public key)

$$\mathbf{b} = \mathbf{A}^t\mathbf{s} + \mathbf{e}$$

$\longleftarrow$

(ciphertext ‘preamble’)

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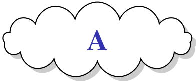

$\mathbf{x}$

$\mathbf{s}, \mathbf{e}$

$$\mathbf{u} = \mathbf{A}\mathbf{x}$$

→ (public key)

$$\mathbf{b} = \mathbf{A}^t\mathbf{s} + \mathbf{e}$$

← (ciphertext ‘preamble’)

$$\boxed{b' = \langle \mathbf{u}, \mathbf{s} \rangle + e'}$$

(key / ‘pad’)

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 $\mathbf{x}$

$\mathbf{s}, \mathbf{e}$ 

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$$\langle \mathbf{x}, \mathbf{b} \rangle \approx \langle \mathbf{u}, \mathbf{s} \rangle$$

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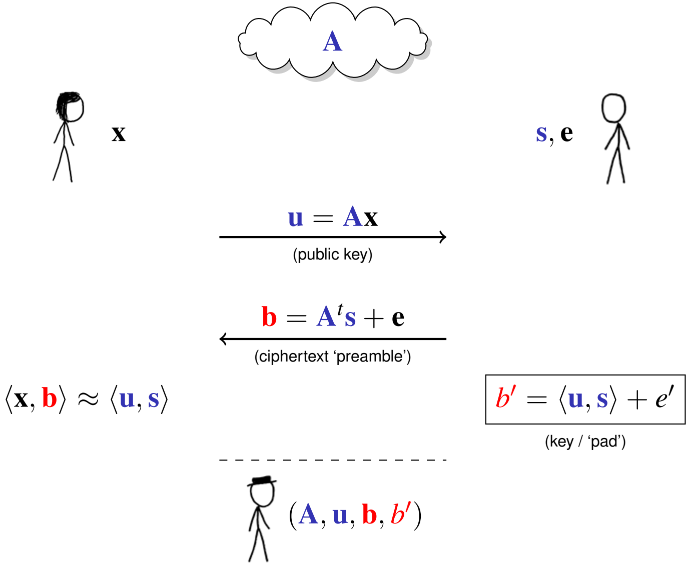

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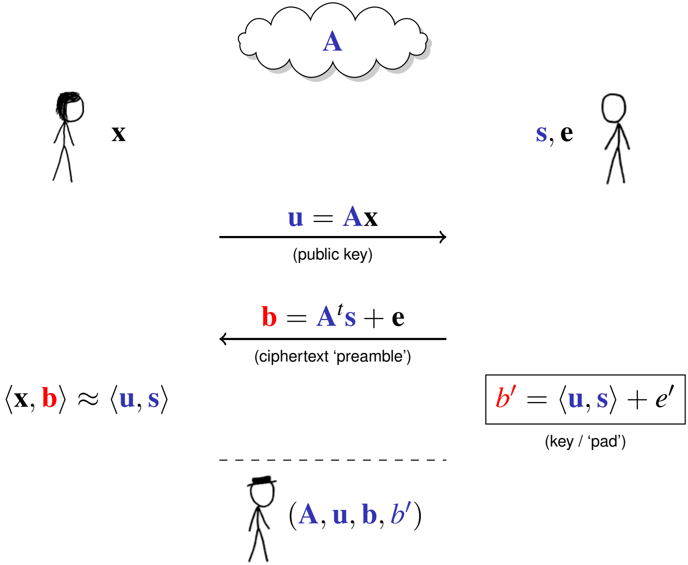
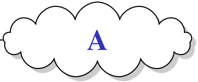


ID-Based Encryption [GPV08]

$\mathbf{x} \leftarrow f_{\mathbf{A}}^{-1}(\mathbf{u})$



$\mathbf{s}, \mathbf{e}$

$\mathbf{u} = H(\text{“alice”})$

(public key)

$\mathbf{b} = \mathbf{A}^t\mathbf{s} + \mathbf{e}$

(ciphertext randomness)

$\langle \mathbf{x}, \mathbf{b} \rangle \approx \langle \mathbf{u}, \mathbf{s} \rangle$

$b' = \langle \mathbf{u}, \mathbf{s} \rangle + e'$

(key / ‘pad’)

Some Open Areas

1. Hash-and-sign sigs / IBE without random oracle ?

   RSA / pairing-style 'accumulator' ?

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3. Connections to number-theoretic problems ?

Further Reading

- Survey "*Cryptographic functions from worst-case complexity assumptions*" [Micciancio07]
- Survey "*Lattice-based cryptography*" [MicciancioRegev09]

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Thanks!