

Exploring Digital Quilt Design Using Manipulatives as a Math Learning Tool

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Introduction

Learning about fractions – how to compare, reduce, represent, write, or draw them – can be a difficult undertaking. Manipulatives enable children to grasp difficult math concepts both literally and figuratively. Pattern tiles, fraction sticks, and fraction pies are common manipulatives used in elementary math to help with these difficult concepts. Many children use pattern tiles to make interesting designs. Through play, they learn about the relationships between some shapes and how to fit them together. Manipulatives can work well as a learning tool for math because they offer a tangible, movable, physical representation that can be used to help learners visualize math concepts in a concrete way. But their affordances for learning are not always well-realized, either because something goes by unnoticed, the play children do with them isn't part of what the teacher supports, or students do not have the manipulatives available when they want to play. Books available on the market (e.g., Sehi-Smith, 1997) make some attempts to enhance learning with pattern tiles by providing structured activities for learners, and some manipulatives have been programmed for use on computers (MATTI, 2001; Mankus, 2001). While they are mathematically interesting and fun to use, they do not focus on integration into the curriculum in formal ways that could reach all students and motivate productive play.

How can we incorporate math manipulatives into the curriculum in such a way that they promote understanding, foster creativity, and perhaps support other curricular areas for more children? This is the question we wish to answer in the DigiQuilt project. Constructionist (Papert, 1991) and Learning by Design (Kolodner et. al., 1998, 2002) approaches both suggest taking a design approach (i.e., asking students to design patterns using manipulatives). Both suggest, as well, that a design approach affords learning not only the math concepts needed for designing but design skills and those skills involved in completing a project. We propose that patchwork and mosaic design projects are a plausible method to promote understanding of math concepts (such as fractions, area, and perimeter) and art concepts (such as symmetry and balance). Further, we propose that virtual manipulatives¹ provide some extra affordances for learning that physical manipulatives lack. With this foundation in mind, we are designing a software environment that integrates math and art for learners to design patchwork quilt blocks and a set of challenges and activities to orchestrate the learning of fractions and simple art concepts. This paper will begin by telling about the software and continue by describing results of a preliminary study exploring the plausibility of the medium and integrating the software in the context of a greater learning environment.

DigiQuilt: Describing the system and supporting design decisions

DigiQuilt (Figure 1) is a digital construction kit for designing quilt blocks. Users create quilt blocks by selecting colored shapes from a palette and placing them into a grid. Shapes can fill a whole grid square, or they can fill some fraction of the square ($1/4$ or $1/2$). Users can rotate shapes to be able to fit them together and place them to make patterns. The selection of shapes and their sizes is based on the ratios and shapes used in Froebel's gifts (Froebel Web, 2001). For ease of

¹ By "virtual" manipulatives, we mean software representations viewed on the screen and manipulated using computer input (e.g. the mouse) rather than in the traditional manner of physical manipulatives.

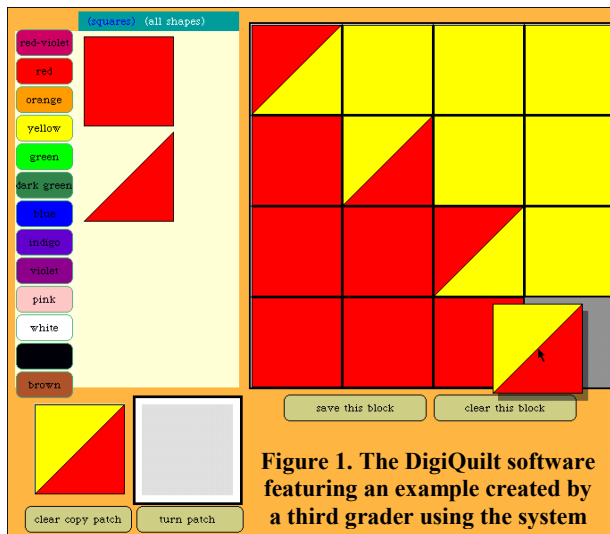
making good-looking designs and talking about the math, all the shapes fit together easily and are sized as whole, half, or one-fourth of the granularity of the grids provided as a framework for the designs. DigiQuilt offers a workspace with a grid, buttons for saving images and clearing the grid, a shape palette with buttons to change their colors, a facility for rotating shapes, and a facility for saving grid patterns so that they can be easily repeated. DigiQuilt has the learner move the shapes to create the designs; much like a physical manipulative.

We have focused on supporting the learning of fractions and symmetry. The standards state that in grades 3-5, students need to learn basic fractions and how to compare them, as well as symmetry and other geometry concepts (NCTM, 2000). Visual art for third graders should include themes like recognizing shapes of the same size or shape, but different position, and symbols of American culture; techniques such as constructing, gluing, manipulating, and stitching; and products such as designs, computer art, and quilts (GDOE, 1999). By manipulating pieces, patches, blocks, and quilts, the learner can explore mathematical concepts in an artistic way. Design activities may help the learners see that a variety of solutions can achieve a single challenge (i.e. there are many ways to arrange patches in a quilt block so that it is $\frac{1}{2}$ one color and $\frac{1}{2}$ another), and that some designs can be used to solve several challenges.

Two features of DigiQuilt were designed specifically to afford the learning of the listed standards. First is the shape rotator. By presenting each shape on the palette in only one orientation and requiring students to rotate them to fit their needs and solve the challenge, the computer offers the affordance that the learner will learn to recognize the same shape turned in different ways. With a physical manipulative, where shapes are scattered on the floor or table, it is possible to select a shape without this recognition. Second is the pattern repetition tool that allows users to copy a group of pieces that make up a patch (a grid-size square) and repeat the design in multiple grid squares. Using this feature saves time when designs use repeating elements. The hypothesis is that learners using DigiQuilt will be able to create comparable designs more quickly than with physical manipulatives and that they will be able to complete more complex designs in a given time than they could using physical manipulatives. The ability to complete more complex designs or to make more designs should allow for deeper learning of the material, either through stepping through a more complex design process or through repeated simple designs that focus on the variety of possible solutions for a single challenge (e.g., creating several representations of one-half that are symmetric from side to side).

We see several other potential advantages of digital manipulatives over physical ones. Constructionism suggests that engagement, and thus learning, is particularly efficacious when learners can design personally meaningful artifacts to share with an audience (Papert, 1991). Artifacts designed on the computer can easily be made permanent and printed out or “published” for display and sharing. Some students might, in addition, want to sew quilt blocks based on their designs, but for those who don’t sew, their designs are captured and available for sharing anyway. The computer medium makes display, sharing, and saving easy.

Eisenberg and Eisenberg (1998) note that using craft media for learning math motivates students because students get to participate in artistic, personal expression, and because of the possible longevity of the artifacts, which can often be displayed publicly. We predicted that because



third graders enjoy art projects so much, activities that involved math and design together would provide a steppingstone from art to math for most, and from math to art for some.

Learning By Design informs us on some ways that we might orchestrate classroom activities to encourage deep learning of process and content through design projects (Kolodner et al. 1998, 2002, Kolodner & Nagel, 1999). First, LBD tells us to make sure students understand the activity they are working on before introducing them to the computer. Second, it tells us that students need a “messaging about” phase before addressing a challenge that promotes the asking of questions important to its solution. Third, LBD suggests orchestration that includes: (1) frequent, authentic, and reflective use of practices over a variety of contexts, both publicly and individually; (2) frequent presentations to peers of results, ideas, and experiences in a context in which peers’ results matter; (3) classroom discussions that summarize and contrast over what’s been presented, making connections between experienced phenomena and targeted concepts; (4) iteration towards solutions to project challenges such that better understanding of concepts is also promoted.

Our intention in integrating DigiQuilt into the classroom is thus to begin with introductory activities using physical manipulatives, much like the “messaging about” phase in LBD, and to present an overall challenge and then a series of smaller challenges. For each design challenge, the learner will work in a pair or small group using either DigiQuilt or physical manipulatives. After several design challenges are met, the small groups will get together to share their designs and describe their solutions. Discussions guided by the teacher will involve students actively presenting ideas and explaining different ways of understanding.

The Study

Is patchwork a plausible medium for learning about fractions and symmetry? How can we orchestrate activities using virtual manipulatives to promote learning through design? What can using physical manipulatives tell us about designing virtual ones? These are the specific questions we set out to answer with this study. In order to understand more about the plausibility of using patchwork design to learn math, and, more specifically, a digital version of patchwork design as a medium for understanding math, we worked with students in a classroom setting using DigiQuilt and a paper version of the manipulatives. To ascertain students’ interests and capabilities, we administered a content pretest and a math and art interest and experience survey. After some introductory activities, we separated the students into groups and moved on to more mini-challenges and discussions, videotaping the interactions for further analysis.

First the pretests were administered. Introductory activities included reading the overall challenge, working together on a mini-challenge, discussing possible solutions, and reading a storybook together about a quilt maker. After completing the introductory activities, students were divided into two groups based on three things: math ability, according to the homeroom teacher, and reading group and homeroom to disrupt class as little as possible. One group used DigiQuilt and the other used paper. Within each group, we assigned pairs or triples to work together. The students using DigiQuilt each used their own computer rather than pairing them up on one machine so that they would work in somewhat the same way as students using the paper version (in parallel rather than strictly on the same designs). The groups used their media in a similar manner to complete several design challenges involving either symmetry or fractions. Often the DigiQuilt and paper pairs worked on the same challenges, and, when the designs were complete, the small groups came together to discuss their designs and describe how their solutions met the challenge. The discussions, artifacts produced, and interactions during the process used to complete the challenge, informed ideas about what worked well and what might be changed to improve both the orchestration and the software. The following section of this paper tells the story of the study in more detail.

Actual enactment and results

Pine Grove Elementary School² is located in a suburban area and had 45% of students qualify for reduced or free lunch in the 2000-2001 school year (OEA, 2001). The two teachers whose classes participated in the study each have about twenty students in class. Each classroom has one computer and the students have about an hour in the computer lab once every six school days. Initially a total of sixteen students brought back permission forms, and those students' experiences will be the focus of the remainder of this paper.

The pretests and introductory activities were completed on three different days to accommodate all of the participants. The content pretest gave us an idea what the students already knew about symmetry and fractions, while the survey told us how they feel in general about math and art. On the art and math interest survey, perhaps the most surprising result was how the kids felt about art vs. math. This group of third graders like math better, think it's more fun, and are more proud of their math assignments. They feel equally creative in art and in math, plus they think math is easier! Also, a surprising number of third graders agreed that there was indeed more than one way to solve a math problem. Later as they solved challenges, there weren't any conflicts about there being more than one acceptable answer (we predicted that they would not agree with answers different from their own for the same challenge).

After the tests, we read them an "overall" challenge (inspired by Kafai & Harel, 1991):

"Second graders need your help to learn about symmetry and fractions. Your challenge is to make examples of different kinds of symmetry and specific fractions, but there is one more constraint: the teacher wants to use the examples to decorate the classroom for the year! That's why we're going to use patchwork quilts. Each block in the quilt will represent a type of symmetry or a particular fraction.

"You'll be solving smaller challenges along the way, and you'll be able to help each other. Some of you will be using the computer and some of you will be using paper to design quilt blocks. You'll be learning some things about symmetry and fractions, and you'll be helping each other along the way. We don't know what will be easier with the computer and what will be easier with the paper, so feel free to tell us what you think. We've never asked third graders to design things to help second graders before, so anything you can tell us about what you're doing along the way would be super."

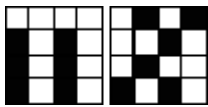
The students had access to a bin of paper shapes and a stack of papers with 16-patch-grids. We told them a little bit about symmetry and gave them an example of a symmetrical design. Then, we asked them to find the lines of symmetry. Sarah was the first to offer a guess about where the line was. She seemed to think that a line of symmetry was supposed to be one of the columns of squares in the grid. We asked if everyone agreed with that attempt. When Gregory said, "No," we asked him to elaborate. He said, "Well, if you fold it there, the green one gets matched up with this [yellow] one." We continued discussion until each student had a chance to share his or her understanding of the lines of symmetry and the group could confirm or redirect the conversation in a productive way.

We then read them their first mini-challenge: "Design a quilt block with at least one line of symmetry." Symmetry was a topic they had a tough time with on the pretests, and we wanted to see right away if they could learn through the process we planned. After about 10 minutes, each child shared one design with the group and showed us the lines of symmetry. Several of the students had large collections of designs (Kelly had about 7 and was proud of the quantity of her designs). We had several discussions like the one mentioned above during each group's introductory activities.

After the introductory mini-challenge and discussion, some of the students wanted to know how they had done on their pretests. On the pretests, an interesting phenomenon stood out. When asked on the pretest to compare the amount of black in two quilt blocks using the words greater than, less than, or equal, 14 of the 16 chose the correct words. On a similar problem with symbols

² Names of school, teachers, and students have been changed to preserve confidentiality.

instead of words, only 10 of the 16 answered correctly, and when asked to place the correct symbol ($<$, $>$, $=$) between $1/2$ and $2/4$, only two out of the 16 answered correctly. This suggests that many students are not making the connection between the concrete and abstract representations of the fractions and symbols: here was an opportunity to connect the manipulatives to solving a problem. We told the students how, on the test, most of them correctly indicated that the two blocks (shown above) have the same amount of black in them, and how that problem is a lot like the later one on the test asking them to compare fractions with different groupings or denominators. To help them see another way to look at it, we used two different arrangements of shapes to try and illustrate the similarity.



As we moved the shapes (grouping them as shown to the left) and described what we were doing, the kids looked confused. After a couple of times moving the shapes we heard, “Ooooooh. I get it. It’s the same *amount* so it’s the same *fraction!*” We asked Abigail to show us what she meant. Abigail took the shapes and moved them in the same way she had observed, and tried to explain it to the other kids.

To wrap up the introductory activities, we read a beautifully illustrated storybook about a quilt maker who makes quilts for the poor and a king who wants to have one. Each page has another example of a quilt block pattern as a detail in the illustration. The students also enjoyed looking through patterns in another book about how to sew patchwork quilt blocks. Later, students were excited to see some in-progress quilt blocks. All these activities contributed to the authenticity of the artifacts for the learners.

The next times students worked on their challenges, they were pulled 4 or 5 at a time from their afternoon activities 1-2 times for a total time of 30-80 minutes. After they settled in to their assigned area, each pair was given an index card with a mini-challenge written on it. Often, the paper and DigiQuilt pairs worked on the same challenges (not necessarily in the same order) so they would have interesting things to talk about together. Some mini-challenges were “easy”: “Make a quilt block that is half one color and half another color,” or, “Make a quilt block with one line of symmetry”; and some were harder: “Make a quilt block that is $1/2$ yellow, $1/4$ blue, and $2/8$ red,” or, “Make 2 quilt blocks to compare $1/2$ and $1/4$ ”. The students were told that they should work together, and, on particularly difficult challenges, it was noted that, “This one might be sort of tricky, so you’ll probably want to talk about how to work it out together.” Usually, by working together with their partners, students were able to complete mini-challenges without asking for much guidance (if any). Sometimes, though, the students were really stuck and needed help.

When students were stuck, usually it was the difficult fractions in the challenge that confused them. One scaffold that seemed to help them understand the fractions was enhancing the grid to help learners look at the workspace in a different way. One challenge we gave Holly and Daniel was, “Make a quilt block that is $1/2$ yellow, $2/8$ red, and $1/4$ blue.” We overheard Holly explaining how to figure out $1/8$, but she was getting it confused with $1/4$. Daniel didn’t understand her attempts to help. He explained what he was doing, and it became clear that he was not thinking about breaking the block up into three pieces that would add up to one whole. “Let’s look at the half first.” Drawing a dark line through the middle emphasized another way to divide the grid. “See how this is still the same block? How many small squares are on each side?” We led him through $1/4$ and $1/8$, drawing lines to emphasize different groupings, and let him notice how $2/8$ and $1/4$ were the same. He seemed quite proud to have figured that out.

Another technique that worked for two students was helping them figure out the fraction first and then make the design, either by giving them the exact number of pieces needed or by helping them set up a work area to gather a set of shapes themselves that fit the fraction. Rosa was confused by the challenge that asked her to fill the grid with $1/4$ red, $1/4$ blue, $1/4$ green, and $1/4$ yellow. The fraction $1/4$ seemed clear to her as she placed one square of each color together in one of the

quadrants. “I’m confused. Is this right?” “Well, so far the area you have filled in is $\frac{1}{4}$ of each color, but can you fill in the whole grid and keep it that way?” She raised an eyebrow. “You mean, like, fill in the whole grid?” We reminded her of the time when she made a design that was $\frac{1}{2}$ pink and $\frac{1}{2}$ blue and how she knew it was $\frac{1}{2}$ of each color even when the pieces were arranged differently. Showing her how to set up the shapes in a work area to figure out the fraction part helped her go on to create a complex design by setting the shapes in one grid in a simple way and using those shapes in her actual design on another grid.

Some of the kids wanted to make more complex designs, but could not imagine how. Pictures from the books mentioned earlier worked as a scaffold for some of the students, motivating them to recreate interesting designs. Holly was particularly interested in recreating designs from pictures. Most of the designs required some translation (from 25- to 16-patch) or lacked lines clearly indicating which shapes to use. Because she found the designs appealing, she was motivated to create them in her own way; and, because they were well-defined designs, we were able to talk about the math.

Some students had difficulty visualizing folding their designs to check for symmetry. Stepping through each patch to see if they matched up could be confusing. Checking for symmetry was easier with the fabric quilt blocks. The students were actually folding the blocks and holding them up to the lights, checking to see if the pieces matched up. With the paper version, that creases the paper and with DigiQuilt, there is no tool specifically designed for that purpose. This seemed to be an engaging and enlightening way to explore symmetry using a physical object.

Discussion

In general, getting started on the computer seemed slightly less intuitive than getting started with the physical manipulatives, which could be manipulated and used right away in some capacity. As far as time needed per design or number of designs produced, there were no noticeable differences in number or complexity of designs created in the same amount of time. Students in both groups found the same challenges difficult and many of the designs produced were similar. However, none of the students in the paper group created designs using two or more triangles in each patch in the same orientation for all 16 patches, which points to the use of the copying tool.

Students using the paper version often expressed interest in trying out the computer, but the reverse was not true. Designs created on the computer masked issues with dexterity since there was no glue or tedious alignment involved. Students printing computer designs talked more about who would receive their designs as a “gift”. Almost every child in the computer group expressed interest in keeping their design printouts, and some requested multiple copies.

The experience of helping third graders learn fractions using physical and virtual manipulatives for design projects suggests some scaffolding that should be added to the software. One useful tool to add might be a work area for the students to gather shapes initially to figure out the fractions or other math content and which shapes they can use to solve the challenge, thus enabling them to make interesting designs with more assurance that the math makes sense. Another useful tool to add to the software might be a “different grid, same problem” scaffold where the learner could select different sets of lines to overlay on their work in progress or a work area to divide the space (group the pieces) in a different way. Some prompting from a teacher or peer to point out the utility of the tool may help the learner look at the problem in terms of understandable fractions.

All of the students were able to understand the easier challenges and create interesting designs to achieve them. With some scaffolding, most students were able to understand the difficult fractions. In both groups, when students struggled to understand the math needed to solve the challenge, they were less inclined to continue on with that challenge and make a more interesting design. We need to make the math-learning fall out of the design process more intuitively to

prevent this sort of frustration. The tools we propose may help ease this frustration.

Merging art and math through design projects (specifically using patchwork) thus seems to be a plausible medium for learning about symmetry and fractions. This study also suggests that while digital or virtual manipulatives offer some affordances that the paper or physical manipulatives may lack, the physical version is still particularly useful for getting started and helping students with specific (unanticipated) learning issues. The physical version of the patchwork environment helped uncover what the software needs to have to be effective as a learning tool, and the software in its current version helped us see some ideas for changes and other directions in the future. Making different tools available to encourage the learner to notice particular concepts using quilt design, this medium might be appropriate for a wide range of ages. Future work is needed on the orchestration and special purpose scaffolding to make patchwork design work for learning about fractions, symmetry, area, perimeter, shapes, color, and pattern.

Acknowledgements

Thank you to the National Science Foundation and the Georgia Institute of Technology for the funding that made this research possible, to the research participants, and to our colleagues at the Georgia Institute of Technology who gave us feedback on this paper.

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