Questions for the Graphics, Animation, Rendering, Modeling Qualifier

Spring 2012

GENERAL QUESTIONS

1) **Quads**: Rendering quads as bi-linear surface patches is expensive, but the alternative of replacing each quad by 2 triangles is ambiguous. We propose to replace each quad by a fan of 4 triangles around a new central vertex V. Explain how to compute the precise location for V that minimizes the total surface area of the four triangles. If a closed-form expression exists for V, please include its derivation or the justification of its correctness. If not, propose an efficient algorithm for computing it and argue its correctness. *Hint: We are mostly interested in your ability to present a mathematically correct argument that justifies the correctness of a solution.*

2) **Ricochet:** You are at point S and you want to shoot a laser beam at a point E that is not visible from S. There are two mirrors $M_1=(P_1,N_1)$ and $M_2=(P_2,N_2)$ in the room, each one defined by a point P_i and a normal N_i. You can see in M_1 the mirror reflection of the mirror reflection in M_2 of E. (In other words, your laser will have to bounce off M_1 first, then M_2, to finally hit E. Explain how to compute the exact direction T of your beam. Discuss why such a tool could be useful in rendering.

3) **Clipping**: The clip C of a triangle T is its intersection with the viewing frustum F. First, prove/disprove that C is convex. Second, indicate the largest possible number of edges in C. Third justify/prove your result using formal arguments.

4) **Falling**: The free fall motion P(t) of a projectile I subject to a constant, but unknown acceleration vector G (which needs not be vertical). Its position has been observed at 3 moments: P(3)=A, P(5)=B, P(7)=C. Explain how to predict its position D at time t=9. In particular, prove or disprove the formulation D=B+2AC, where AC denotes the vector from A to C. If it is false, strive to provide the correct formulation that is similar to this one and prove its correctness.

5) **Morph**: You are given two triangle strips of the same connectivity, but different shapes. Explain how you would compute a "good" morph between them. List several reasonable goodness measures for such a morph (justifying where and they may be desired) and measure your solution against them. Provide the details of how exactly you would compute the morph.

6) **Edge motion**: Consider four consecutive key frames (A_1,B_1), (A_2,B_2), (A_3,B_3), (A_4,B_4), showing an edge in 3D. (a) Explain, in detail, how you would compute a **smooth** animation of the edge that **interpolates** these key-frames. (b) Point out the drawbacks of your solution (by providing an example where the solution does not yield satisfactory results) and suggest other (possibly more complex) approaches.

MODELING

1) **Rooms**: You are given a table G[Nv] of vertices and a table V[Nt][3] of vertex references, where each entry V[t] is a table of the 3 indices to the vertices of triangle t (in no particular order). The mesh topology represents correctly its embedding, meaning that all cells (vertices, relatively open edges, and relatively open triangles are pairwise disjoint). The mesh represents a non-manifold surface S (the union of the cells). Explain how you would compute the number of rooms (maximally connected components) of the complement of S. One of the rooms is infinite. Hint: Provide a high level description of the principal idea and then details of the non-trivial algorithms. If you assume a specific data structure or operators, please provide details about how what they are and how they could be computed.

2) **Knots**: You are given 2 polyloop curves, A and B, in 3D. Each one is represented by a circular list of vertex locations. You are told that each one is **simple**, meaning that it may be deformed into a circle by a continuous deformation without crossing itself or creating non-manifold points. We say that A and B are **free** if they can be both deformed into separate (non-linked circles) by a continuous deformation without any collisions. Propose a constructive algorithm for testing whether A and B are free. By constructive, we mean that we do not want a solution that simulates the progressive contraction of the two curves into small and disjoint circles while avoiding collisions. Instead, we want a geometric test that computes something and answers the question. If you can think of such a test, describe the algorithm, provides its asymptotic complexity, and justify its validity using a formal proof or at least an intuitive argument. If you think that no such test exists, argue why.

3) **Genus**: Triangle mesh M represents the triangulation of a connected, orientable, and manifold surface. It is represented by a corner table. You are given the number Nv of its vertices and the number Nt of its triangles. However, M may have one or more border loops around its holes. Explain how you would compute its genus. *Hint: First, define the genus and explain your approach at the high level. Then provide details of key data structures and of non-trivial algorithms, as time permits. For example, if you are using an intermediate mesh representation, explain what it is, how you construct it from the input format, and how you process it. Give the asymptotic complexity of the whole process.*

4) **Inbetweening**: You are given two convex polyhedral solids A and B, each represented by a triangle mesh. (The two triangle meshes may have different numbers of vertices and different connectivity graphs.) The two solids are not registered and may be far away from each other. We want to compute an animated inbetweening model, I(A,t,B), that is a family of polyhedra parameterized by time t, that is interpolating (meaning that I(A,0,B)=A, I=(A,1,B)=B), that is a continuous function of time, and that ensures that the evolving shape has a valid triangle mesh representation that is free from self-intersections and remains convex. Provide a definition of your solution, a description of its representation, an outline of the algorithm or computing it, and the asymptotic complexity of that algorithm. Discuss limitations of your solution and suggest avenues for addressing them.

ANIMATION

1) **A hybrid system**: Golem Krang is a robot designed for mobile manipulation. It has a wheeled base and a humanoid torso for both navigation and manipulation. You are asked to write a simulation program for Golem Krang. Because the wheels are fully actuated, the horizontal translation and yaw rotation of the root can be treated as kinematic motion following a desired trajectory. However, the degrees of freedom on the torso need to by simulated dynamically and should react to the acceleration of the root accurately. Here is a proposed solution:

1. Compute the acceleration from the desired kinematic motion using finite differences

2. Multiply the acceleration by the total mass to get force

3. Apply force to the root

   Is this correct? Explain your answer. If it's not correct, provide a correct solution.

2) **A constrained system**: The figure below-left is a 2D particle system with constraints. One particle (A) can move freely on the circle but cannot be pulled off of it. The other particle (B) is connected to A by a fixed length rod. We want to simulate this system in the generalized coordinates instead of the Cartesian coordinates. What is the number of degrees of freedom in the generalized coordinates? Please write down the equations of motion. Let us now add a third particle (C) that is restricted to slide on the circle and is connected by a fixed length rod to A (right figure). How many degrees of freedom does the system have now? Explain your answer.
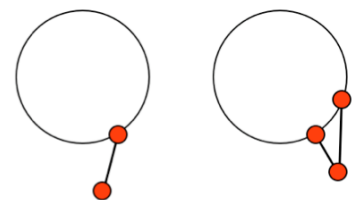
3) **Force conversion**: You are given a two-linked 3D pendulum with a ball joint represented as an exponential map and a universal joint represented as two Euler angles. You develop your own code to compute control forces in the generalized coordinates, but you rely on Open Dynamic Engine (ODE) for forward simulation. The problem is that ODE applies forces and torques in the Cartesian space, not in the generalized coordinates. Let the current control force be $Q \in R^5$, can you convert it to Cartesian forces and torques?

ball joint: exponential map

universal joint: euler angle
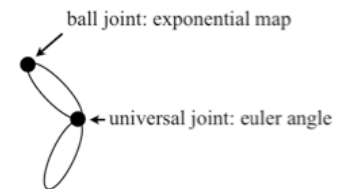
4) **Control force**: To assemble IKEA furniture, BESTÅ, I need to screw a nail into a wooden bar using my right hand while holding down the bar using my left hand. How do I command my joint actuators to achieve such a task? How do I avoid interference between these two tasks?

RENDERING

1) **Museum Jewelry**: You are in charge of creating a virtual museum for a newly discovered trove of 16th century jewelry, including gemstone necklaces, bracelets, and earings. Your task is to create a rendering system that is capable of synthesizing images of the jewelry pieces that are made of gems and metals. Describe the rendering techniques that you would use in order to synthesize images that include reflection and refraction, caustics, and anisotropic surface reflection due to grooves in the brushed metal pieces. Your system should **not** make use of image-based rendering.

2) **Four Corners**: Describe how a rasterization renderer estimates the texture footprint of a pixel in the final image. (The footprint is the texture-space region over which filtering is performed for a given pixel.) Also explain how this information can be used to anti-alias the texture. Part of your answer should include a description of the reconstruction kernel that is being used. Next, consider how such a texture footprint could be created in a ray tracer. Specifically, assume that we shoot four rays per pixel – one for each corner of the pixel. Explain how using information from the four corner rays per pixel can be used to produce anti-aliased textured images. How could you adjust your algorithm to work when less than four of the rays actually hit the textured object in question?

3) **Render Wars**: There is an ongoing battle between ray tracing renderers and rasterization renderers in terms of features. Describe the approaches that these two kinds of rendering systems take to produce (or approximate) the following effects. Compare each pair of approaches (ray tracing versus rasterization) with respect to image quality and speed: a) Hard-edged shadows, b) Soft shadows (penumbra), c) Mirrored reflections, d) Fine detailed bumpy surfaces.

4) **Virtual Tech**: A wealthy alum has donated a large amount of money to Georgia Tech for the purpose of preserving a view of the school as it is today. You have been given the task of accomplishing this using an image-based rendering technique of your choice. Specifically, you must select one of these three approaches to creating a Virtual Tech: Light Field Rendering, Lumigraph, or Concentric Mosaics. Select one of these methods, and describe how you would create a Virtual Georgia Tech that allows a virtual visitor to see the way the campus looks from a number of fixed locations using this technique. Your description should include how the image data is to be captured, what is the form of storage of the data, how you might compress the data, and how you would create images from the data. In addition, outline a method for being able to display images that smoothly move the view between pairs of locations on the campus (visually connected corridors).