Graphics QUAL spring 2018 (oral)

Please prepare an oral presentation of answers to the following 3 questions. To save time, in your presentation, you may use some slides to show math derivations, figures, algorithms, or results. But you will be expected to explain/argue the most interesting parts of your solutions on the white board and answer questions about justifications, limitations, or extensions of your solutions. If you find that some of the questions are ambiguous or incoherent, please decide, yourself, on the best interpretation (the one that you think is the most interesting and appropriate for this qualifier).

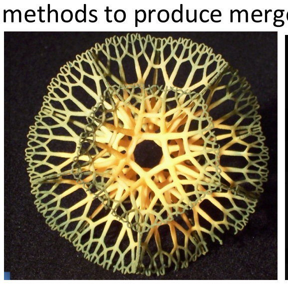
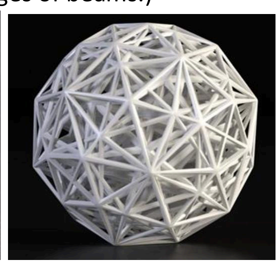
Q1: MORPHING TRIANGLES

Consider two arbitrary triangles, (A_0, B_0, C_0) and (A_1, B_1, C_1), in 3D with a given vertex correspondence.

(1) Explain at a high level how to compute the "best" morph between them.
(2) Explain in details how to compute the vertices (A_t, B_t, C_t) of the animated triangle for the current time t.
(3) Briefly discuss other options and argue why the one you chose is the best.

Q2: MERGING BEAMS

You are given a collection of 3D points ("joints") and a set of edges that each connects a pair of these joints so as to form a lattice. In order to 3D print such models, your task is to turn each edge into a tube-like (beam) shape that is bounded by polygonal faces and also to compute a clean merge where these beams meet. You are not allowed to use the general CSG-to-boundary conversion algorithm. Instead, you should exploit the ambiguities in the above specification to produce a different solution that yields polygons that merge together the beams in a clean manner. The final result must be a single watertight polygonal mesh. State your assumptions and analyze your algorithm in terms of robustness, ease of programming, and aesthetics of the final results. You should strive to provide a solution that is pleasing on all three aspects. Present the details: data structure used, geometric computation and its robustness, algorithm in easily readable pseudo-code, complexity, limitations, and justification of correctness. (See images below for 3D printed models that use various methods to produce merges of beams.)


Q3: CURVED BEAMS

The goal here is to define a **beam** with a **curved axis**, to propose practical **control parameters**, and to provide the mathematical and algorithmic construction of its **intrinsic parameters** and of its **bounding surfaces**. You are given two balls, one with center A and radius a and one with center B and radius b. We define the beam as the infinite union of a parametric ball of center C(t) and radius r(t) for t in [0,1]. Let C be the curved axis of the beam: I.e., the union of all points C(t).

(a) Assume that C is a **log-spiral** (hence planar) curve and that ball(C(t),r(t)) is a **log-spiral motion**. Assume that C interpolates points A and B (i.e., C(0)=A and C(1)=B) and function r interpolates the corresponding radii (i.e., r(0)=a and r(1)=b). (1) Provide the precise mathematical expression of C(t) and of r(t). (2) Explain how many degrees of freedom there are to this construction in 3D. (3) Explain which control parameters you suggest to expose and why. (4) Explain how to compute the spiral parameters from these control parameters.
(b) Explain how to define the **surface** S that contains the non-spherical (tubular) portion of the boundary of the beam. (1) Provide a precise mathematical formulation of S. (2) Discusses how S relates to mathematical models of similar surface previously studied in prior art. (3) Suggest an approach for testing whether the tubular portion of S that bounds the beam is self-crossing.
(c) Extend your answers for (a) and (b) to the situation where C is **not planar**, but a conchospiral curve, and where ball(C(t),r(t)) follow a **swirl motion**.