1 Gaussian Process Regression

Stranded on a desert island, the key to your survival is one last regression problem. Fortunately, you brought your Gaussian Processes reading on your ill-fated flight, and even more fortunately the plane did not have in-flight entertainment or wifi so you studied it in great detail this time. Given the data \( x = \{-2, 1, 2\} \) and \( y = \{0.5, -2, 1\} \), you need to come up with a mean estimate for arbitrary locations \( x^* \in [-8 \ldots 8] \). You may plot variance estimates if you wish, but are not required to. Assume a prior mean of 0 on all \( y \) values, and assume some noise. Hint: We are not looking for exact answers, your survival depends on your ability to reason about and understand kernel functions, not invert 3x3 matrices.

a) RBF/Squared Exponential Kernel: \( k(x, x') = \exp\left(-\frac{||x-x'||^2}{\ell^2}\right) \) for the length-scales 1.0, 0.2, and 5.0 (please write each length scale on its plot). (20pts)
b) \( k(x, x') = x^T x' \) (10pts):

![Figure 2: Linear Kernel](image)

\[ k(x, x') = x^T x' \]

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c) \( k(x, x') = 1 \) (10pts):

![Figure 3: Constant Kernel](image)

\[ k(x, x') = 1 \]
d) \( k(x, x') = 1(x == x') \) (10pts):

![Figure 4: Indicator Kernel](image)

**Figure 4: Indicator Kernel**

e) To be a covariance function, or equivalently a kernel function, \( k(x, y) \) must be positive definite. What does this mean? (20pts)

f) What would happen if you tried to parameterize a Gaussian Process in the “natural” parameterization. Can you make something like this work? Why or why not? (30pts)