ANOVA:
Analysis of Variance

An example ANOVA problem

25 individuals split into three between-subject conditions: A, B and C

- A: 5,6,6,7,7,8,9,10 [8 participants, mean: 7.25]
- B: 7,7,8,9,9,10,10,11 [8 participants, mean: 8.875]
- P: 7,9,9,10,10,10,11,12,13 [9 participants, mean: 10.11]

Are the differences between the conditions significant?

What does ANOVA do?

ANOVA tests the following hypotheses:

- H_0 (null hypothesis): The means of all the groups are equal.

- H_a: Not all the means are equal
  - doesn’t say how or which ones differ.
  - Can follow up with “multiple comparisons”

Notation for ANOVA

- n = number of individuals all together
- i = number of groups
- $\bar{x}$ = mean for entire data set is

Group i has

- n_i = # of individuals in group *i*
- x_{ij} = value for individual *j* in group *i*
- $\overline{x_i}$ = mean for group *i*
- s_i = standard deviation for group *i*

How ANOVA works

ANOVA measures two sources of variation in the data and compares their relative sizes

- variation BETWEEN groups
  - for each data value look at the difference between its group mean and the overall mean

$$(\bar{x}_i - \bar{x})^2$$

- variation WITHIN groups
  - for each data value we look at the difference between that value and the mean of its group

$$\left(x_{ij} - \bar{x}_i\right)^2$$

F-score

- The ANOVA F-statistic is a ratio of the Between Group Variaton divided by the Within Group Variation:

$$F = \frac{Between}{Within}$$

- A large F is evidence *against* H_0, since it indicates that there is more difference between groups than within groups.

ANOVA Output for Our Example

Analysis of Variance summary

Source	DF	SS	MS	F	P
Treatment [between groups]	2	34.74	17.37	6.45	0.006
Error [within groups]	22	59.26	2.69		
Total	24	94.00			

ANOVA Output for Our Example

Analysis of Variance summary

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Total	24	94.00			

# of data values - # of groups

(df for each group added together)

1 less than # of groups

1 less than # of individuals

ANOVA Output for Our Example

Analysis of Variance summary

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Treatment [between groups]	2	34.74	17.37	6.45	0.006
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Total	24	94.00			

$$\sum (x_{ij} - \bar{x}_i)^2$$

$$\sum (x_{ij} - \bar{x})^2$$

$$\sum (\bar{x}_i - \bar{x})^2$$

SS ="sum of squares"

ANOVA Output for Our Example

MSG = SSG / DFG
MSE = SSE / DFE

Analysis of Variance summary

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34.74/2 = 17.37

F = MSG / MSE

P-value comes from F(DFG,DFE)

Post-hoc analysis

- ANOVA indicates that the groups do not all appear to have the same means… what next? How do we know what the differences really are?

- If we only had two groups, then we're done, we know the difference between them is significant.
- If we have three or more groups, then a post hoc test is needed to determine which groups are significantly different from each other

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Post-hoc analysis

- Multiple post hoc analysis methods exist
- We most commonly see the Tukey test

- Results for our example dataset:

```
HSD[.05]=2.02; HSD[.01]=2.61
M1 vs M2   nonsignificant
M1 vs M3   P<.01
M2 vs M3   nonsignificant
```

HSD = the absolute (unsigned) difference between any two sample means required for significance at the designated level.

Assumptions of ANOVA

- The distribution of data in each group is approximately normal
  - check this by looking at histograms and/or normal quantile plots
  - can handle some non-normality, but not severe outliers
- Standard deviations of each group are approximately equal
  - rule of thumb: ratio of largest to smallest sample st. dev. must be less than 2:1

Our case study…

- Our case study has many similarities to the above example, but in that case it's a two-way ANOVA. I leave it to you to decide whether that is the appropriate test and what conclusions can be drawn from it based on the way it was conducted.

ANOVA Summary					
A = row variable (Mobile Robot / No Mobile Robot) B = column variable (No Social / Social) Subj = subjects					
Source	Sum of Squares	df	Mean Square	F	p
Subjects	134.4	9			
Within Subjects					
A	592.9	1	592.9	70.5833	<.0001
Subj x A	75.6	9	8.4		
B	115.6	1	115.6	13.7076	0.004902
Subj x B	75.9	9	8.4333		
A x B	160	1	160	17.6686	0.002295
Subj x A x B	81.5	9	9.0556		
TOTAL	1235.9	39			