

Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images

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Abstract—We make an analogy between images and statistical mechanics systems. Pixel gray levels and the presence and orientation of edges are viewed as states of atoms or molecules in a lattice-like physical system. The assignment of an energy function in the physical system determines its Gibbs distribution. Because of the Gibbs distribution, Markov random field (MRF) equivalence, this assignment also determines an MRF image model. The energy function is a more convenient and natural mechanism for embodying picture attributes than are the local characteristics of the MRF. For a range of degradation mechanisms, including blurring, nonlinear deformations, and multiplicative or additive noise, the posterior distribution is an MRF with a structure that can be modeled. For each mechanism, there is a Markov field distribution that can be used to generate the degraded image. This provides a simple approach for simulating the degradation process. By further modeling the Markov field distribution, a posterior distribution can be sampled by means of stochastic relaxation. For each mechanism, a sequence of images can be generated by stochastic relaxation which converges in an appropriate sense to the MAP estimate. This sequence evolves by *local* (and potentially *parallel*) changes in pixel gray levels and in locations and orientations of boundary elements. Deterministic, iterative-improvement methods generate a sequence of images that monotonically increase the posterior distribution (our “objective function”). In contrast, stochastic relaxation permits changes that *decrease* the pos-

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