Class Today

• Project 1 is going out today
• Read Szeliski 2.1, especially 2.1.5
• More about projections
• Light, color, biological vision
The Geometry of Image Formation

Mapping between image and world coordinates

– Pinhole camera model
– Projective geometry
  • Vanishing points and lines
– Projection matrix
What do you need to make a camera from scratch?
Let’s design a camera

– Idea 1: put a piece of film in front of an object
– Do we get a reasonable image?
Pinhole camera

Idea 2: add a barrier to block off most of the rays
- This reduces blurring
- The opening known as the aperture
Pinhole camera

\[ f = \text{focal length} \]
\[ c = \text{center of the camera} \]
Camera obscura: the pre-camera

- Known during classical period in China and Greece (e.g. Mo-Ti, China, 470BC to 390BC)

Illustration of Camera Obscura

Freestanding camera obscura at UNC Chapel Hill

Photo by Seth Ilys
Camera Obscura used for Tracing

Lens Based Camera Obscura, 1568
Accidental Pinhole and Pinspeck Cameras
Revealing the scene outside the picture.
Antonio Torralba, William T. Freeman
Accidental Cameras

a) Input (occluder present)  b) Reference (occluder absent)

c) Difference image (b-a)  d) Crop upside down  e) True view
First Photograph

Oldest surviving photograph
  – Took 8 hours on pewter plate

Joseph Niepce, 1826

Photograph of the first photograph

Stored at UT Austin

Niepce later teamed up with Daguerre, who eventually created Daguerrotypes
Camera and World Geometry

How tall is this woman?

How high is the camera?

What is the camera rotation?

What is the focal length of the camera?

Which ball is closer?
Dimensionality Reduction Machine (3D to 2D)

3D world

2D image

Point of observation
Projection can be tricky...
Projection can be tricky...
Projective Geometry

What is lost?

• Length
Length and area are not preserved
Projective Geometry

What is lost?

- Length
- Angles
Projective Geometry

What is preserved?

• Straight lines are still straight
Vanishing points and lines

Parallel lines in the world intersect in the image at a “vanishing point”
Vanishing points and lines

Vanishing Point

Vanishing Line

Vanishing Point
Vanishing points and lines

Vertical vanishing point (at infinity)

Vanishing point

Vanishing point

Slide from Efros, Photo from Criminisi
Projection: world coordinates $\rightarrow$ image coordinates

If $X = 2$, $Y = 3$, $Z = 5$, and $f = 2$

What are $U$ and $V$?

$$x = \begin{bmatrix} u' \\ v' \end{bmatrix}$$

$$u' = -x \ast \frac{f}{z}$$
$$v' = -y \ast \frac{f}{z}$$

$$u' = -2 \ast \frac{2}{5}$$
$$v' = -3 \ast \frac{2}{5}$$
Projection: world coordinates $\rightarrow$ image coordinates

$$p = \begin{bmatrix} u \\ v \end{bmatrix}$$

Optical Center $(u_0, v_0)$

Camera Center $(t_x, t_y, t_z)$

$$P = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

How do we handle the general case?
Interlude: why does this matter?
Relating multiple views
Photo Tourism
Exploring photo collections in 3D

Noah Snavely  Steven M. Seitz  Richard Szeliski
University of Washington  Microsoft Research

SIGGRAPH 2006
Projection: world coordinates $\rightarrow$ image coordinates

Camera Center $(t_x, t_y, t_z)$

Optical Center $(u_0, v_0)$

$p = \begin{bmatrix} u \\ v \end{bmatrix}$

How do we handle the general case?
Homogeneous coordinates

Conversion

Converting to *homogeneous* coordinates

\[
(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
\]

homogeneous image coordinates

\[
(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\]

homogeneous scene coordinates

Converting *from* homogeneous coordinates

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)
\]

\[
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)
\]
Homogeneous coordinates

Invariant to scaling

\[
\begin{bmatrix}
x \\
y \\
w
\end{bmatrix}
= k
\begin{bmatrix}
xk \\
yk \\
wk
\end{bmatrix}
\Rightarrow
\begin{bmatrix}
xk \\
wk \\
ky \\
wk
\end{bmatrix}
= \begin{bmatrix}
x/w \\
w \\
y/w
\end{bmatrix}
\]

Homogeneous Coordinates

Cartesian Coordinates

Point in Cartesian is ray in Homogeneous
Projection matrix

\[ x = K[R \ t]X \]

- **x**: Image Coordinates: \((u,v,1)\)
- **K**: Intrinsic Matrix (3x3)
- **R**: Rotation (3x3)
- **t**: Translation (3x1)
- **X**: World Coordinates: \((X,Y,Z,1)\)
Projection matrix

Intrinsic Assumptions
- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

\[ X = K \begin{bmatrix} I & 0 \end{bmatrix} X \]

\[ w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]
Projection matrix

Intrinsic Assumptions
- Unit aspect ratio
- Optical center at (0,0)
- No skew

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

\[ x = K [I \ 0] X \quad \Rightarrow \quad w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \]
Remove assumption: known optical center

Intrinsic Assumptions
• Unit aspect ratio
• No skew

Extrinsic Assumptions
• No rotation
• Camera at (0,0,0)

\[
x = K[I \ 0]X
\]

\[
w = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 & 0 \\ 0 & f & v_0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\]
Remove assumption: square pixels

Intrinsic Assumptions
• No skew

Extrinsic Assumptions
• No rotation
• Camera at (0,0,0)

\[ x = K \begin{bmatrix} I & 0 \end{bmatrix} X \]

\[
\begin{bmatrix}
 u \\
 v \\
 1
\end{bmatrix} = \begin{bmatrix}
 \alpha & 0 & u_0 & 0 \\
 0 & \beta & v_0 & 0 \\
 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
 x \\
 y \\
 z \\
 1
\end{bmatrix}
\]
Remove assumption: non-skewed pixels

Intrinsic Assumptions

Extrinsic Assumptions
- No rotation
- Camera at (0,0,0)

\[
x = K \begin{bmatrix} I & 0 \end{bmatrix} X
\]

\[
w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha & s & u_0 & 0 \\ 0 & \beta & v_0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\]

Note: different books use different notation for parameters
Oriented and Translated Camera
Allow camera translation

\[ x = K [I \ t] X \]

Intrinsic Assumptions
Extrinsic Assumptions
• No rotation
3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:

\[ R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \]

\[ R_y(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \]

\[ R_z(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
Allow camera rotation

\[ x = K [ R \quad t ] X \]

\[
\begin{bmatrix}
u \\
v \\
1
\end{bmatrix} =
\begin{bmatrix}
\alpha & s & u_0 \\
0 & \beta & v_0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
 r_{11} & r_{12} & r_{13} & t_x \\
r_{21} & r_{22} & r_{23} & t_y \\
r_{31} & r_{32} & r_{33} & t_z \\
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]
Degrees of freedom

\[ x = K[R \ t]X \]

\[
\begin{bmatrix}
  u \\
  v \\
  1
\end{bmatrix} =
\begin{bmatrix}
  \alpha & s & u_0 \\
  0 & \beta & v_0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  r_{11} & r_{12} & r_{13} & t_x \\
  r_{21} & r_{22} & r_{23} & t_y \\
  r_{31} & r_{32} & r_{33} & t_z
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
  1
\end{bmatrix}
\]
Field of View (Zoom, focal length)

From London and Upton
Beyond Pinholes: Radial Distortion

No Distortion  Barrel Distortion  Pincushion Distortion

Corrected Barrel Distortion

Image from Martin Habbecke
Things to remember

- Vanishing points and vanishing lines
- Pinhole camera model and camera projection matrix
- Homogeneous coordinates

\[ \mathbf{x} = \mathbf{K} [\mathbf{R} \quad \mathbf{t}] \mathbf{X} \]

\[
(x, y) \Rightarrow \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]
Reminder: read your book

• Lectures have assigned readings
• Szeliski 2.1 and especially 2.1.5 cover the geometry of image formation
5 minute break
Image Formation

Digital Camera

The Eye
A photon’s life choices

• Absorption
• Diffusion
• Reflection
• Transparency
• Refraction
• Fluorescence
• Subsurface scattering
• Phosphorescence
• Interreflection
A photon’s life choices

• Absorption
• Diffusion
• Reflection
• Transparency
• Refraction
• Fluorescence
• Subsurface scattering
• Phosphorescence
• Interreflection
A photon’s life choices

• Absorption
• **Diffuse Reflection**
• Reflection
• Transparency
• Refraction
• Fluorescence
• Subsurface scattering
• Phosphorescence
• Interreflection
A photon’s life choices

- Absorption
- Diffusion
- **Specular Reflection**
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection
A photon’s life choices

- Absorption
- Diffusion
- Reflection
- **Transparency**
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection

\[ \lambda \] light source
A photon’s life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- **Refraction**
- Fluorescence
- Subsurface scattering
- Phosphorescence
- Interreflection
A photon’s life choices

• Absorption
• Diffusion
• Reflection
• Transparency
• Refraction
• **Fluorescence**
• Subsurface scattering
• Phosphorescence
• Interreflection
A photon’s life choices

• Absorption
• Diffusion
• Reflection
• Transparency
• Refraction
• Fluorescence
• **Subsurface scattering**
• Phosphorescence
• Interreflection
A photon’s life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering

**Phosphorescence**
- Interreflection
A photon’s life choices

- Absorption
- Diffusion
- Reflection
- Transparency
- Refraction
- Fluorescence
- Subsurface scattering
- Phosphorescence
- **Interreflection**
Lambertian Reflectance

• In computer vision, surfaces are often assumed to be ideal diffuse reflectors with no dependence on viewing direction.
A digital camera replaces film with a sensor array
- Each cell in the array is light-sensitive diode that converts photons to electrons
- Two common types
  - Charge Coupled Device (CCD)
  - CMOS
Sensor Array

**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.
FIGURE 2.16 Generating a digital image: (a) Continuous image. (b) A scan line from $A$ to $B$ in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.
Interlace vs. progressive scan

Progressive scan


Slide by Steve Seitz
Interlace

Rolling Shutter
The human eye is a camera!

- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris
  - What’s the “film”?
    - photoreceptor cells (rods and cones) in the **retina**
Aside: why do we care about human vision in this class?

• We don’t, necessarily.
Ornithopters
Why do we care about human vision?

- We don’t, necessarily.
- But cameras necessarily imitate the frequency response of the human eye, so we should know that much.
- Also, computer vision probably wouldn’t get as much scrutiny if biological vision (especially human vision) hadn’t proved that it was possible to make important judgements from 2d images.
Does computer vision “understand” images?

"Can machines fly?" The answer is yes, because airplanes fly.

"Can machines swim?" The answer is no, because submarines don't swim.

"Can machines think?" Is this question like the first, or like the second?

Source: Norvig
The Retina

Cross-section of eye

- Ganglion cell layer
- Bipolar cell layer
- Receptor layer
- Pigmented epithelium

Cross section of retina

- Ganglion axons
What humans don’t have: tapetum lucidum

Human eyes can reflect a tiny bit and blood in the retina makes this reflection red.
Two types of light-sensitive receptors

**Cones**
- cone-shaped
- less sensitive
- operate in high light
- color vision

**Rods**
- rod-shaped
- highly sensitive
- operate at night
- gray-scale vision
Rod / Cone sensitivity

- Dazzling light; bright sun on snow
- Outdoors in full sunlight
- Outdoors under a tree on a sunny day
- Comfortable indoor illumination; night sports events
- Threshold for perception of color; bright moonlight
- Threshold when dark-adapted

Night Sky: why are there more stars off-center?
Averted vision: http://en.wikipedia.org/wiki/Averted_vision
Wait, the blood vessels are in front of the photoreceptors??

https://www.youtube.com/watch?v=L_W-IXqoxHA