Project 1
Hybrid Images

Why do we get different, distance-dependent interpretations of hybrid images?
Thinking in Frequency
Recap of Filtering

• Linear filtering is dot product at each position
  – Not a matrix multiplication
  – Can smooth, sharpen, translate (among many other uses)

• Be aware of details for filter size, extrapolation, cropping
Median filters

- A **Median Filter** operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?
Comparison: salt and pepper noise

<table>
<thead>
<tr>
<th>Mean</th>
<th>Gaussian</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="3x3 Mean" /></td>
<td><img src="image2" alt="3x3 Gaussian" /></td>
<td><img src="image3" alt="3x3 Median" /></td>
</tr>
<tr>
<td><img src="image4" alt="5x5 Mean" /></td>
<td><img src="image5" alt="5x5 Gaussian" /></td>
<td><img src="image6" alt="5x5 Median" /></td>
</tr>
<tr>
<td><img src="image7" alt="7x7 Mean" /></td>
<td><img src="image8" alt="7x7 Gaussian" /></td>
<td><img src="image9" alt="7x7 Median" /></td>
</tr>
</tbody>
</table>
Review: questions

1. Write down a 3x3 filter that returns a positive value if the average value of the 4-adjacent neighbors is less than the center and a negative value otherwise.

2. Write down a filter that will compute the gradient in the x-direction:

   \[ \text{grad}_x(y,x) = \text{im}(y,x+1)-\text{im}(y,x) \text{ for each } x, y \]
Review: questions

3. Fill in the blanks:
   a) \_ = D \times B
   b) A = \_ \times \_ 
   c) F = D \times \_
   d) \_ = D \times D
Today’s Class

• Fourier transform and frequency domain
  – Frequency view of filtering
  – Hybrid images
  – Sampling

• Reminder: Read your textbook
  – Today’s lecture covers material in 3.4
Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?
Why does a lower resolution image still make sense to us? What do we lose?
Thinking in terms of frequency
Jean Baptiste Joseph Fourier (1768-1830) had a crazy idea (1807): any univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.

• Don’t believe it?
  – Neither did Lagrange, Laplace, Poisson and other big wigs
  – Not translated into English until 1878!

• But it’s (mostly) true!
  – Called Fourier Series
  – There are some subtle restrictions

...the manner in which the author arrives at these equations is not exempt of difficulties and...his analysis to integrate them still leaves something to be desired on the score of generality and even rigour.
How would math have changed if the Slanket or Snuggie had been invented?
A sum of sines

Our building block:

\[ A \sin(\omega x + \phi) \]

Add enough of them to get any signal \( g(x) \) you want!
Frequency Spectra

example: \( g(t) = \sin(2\pi f t) + \frac{1}{3}\sin(2\pi (3f) t) \)
Frequency Spectra
Frequency Spectra
Frequency Spectra
Frequency Spectra
Frequency Spectra
Frequency Spectra
Frequency Spectra

\[ A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi kt) \]
Example: Music

- We think of music in terms of frequencies at different magnitudes
Other signals

• We can also think of all kinds of other signals the same way

xkcd.com
Fourier analysis in images

Intensity Image

Fourier Image
Fourier Transform

- Fourier transform stores the magnitude and phase at each frequency
  - Magnitude encodes how much signal there is at a particular frequency
  - Phase encodes spatial information (indirectly)
  - For mathematical convenience, this is often notated in terms of real and complex numbers

\[
A = \pm \sqrt{R(\omega)^2 + I(\omega)^2} \quad \text{Amplitude:}
\]
\[
\phi = \tan^{-1} \frac{I(\omega)}{R(\omega)} \quad \text{Phase:}
\]
Salvador Dali invented Hybrid Images?

Salvador Dali
“Gala Contemplating the Mediterranean Sea, which at 30 meters becomes the portrait of Abraham Lincoln”, 1976
Fourier Bases

Teases away fast vs. slow changes in the image.

This change of basis is the Fourier Transform
Fourier Bases

in Matlab, check out: imagesc(log(abs(fftshift(fft2(im)))));
Intermission
Slow mo guys – Saccades and CRTs

• https://youtu.be/Fmg9ZOHESgQ?t=21s
• https://youtu.be/3BJU2drBtCM
Man-made Scene
Can change spectrum, then reconstruct
Low and High Pass filtering
The Convolution Theorem

• The Fourier transform of the convolution of two functions is the product of their Fourier transforms

$$F[ g * h ] = F[ g ] F[ h ]$$

• **Convolution** in spatial domain is equivalent to **multiplication** in frequency domain!

$$g * h = F^{-1} [ F[ g ] F[ h ] ]$$
Filtering in spatial domain

\[
\begin{bmatrix}
1 & 0 & -1 \\
2 & 0 & -2 \\
1 & 0 & -1 \\
\end{bmatrix}
\]

[Intensity image] \* [Kernel] = [Resulting image]
Filtering in frequency domain

1. FFT
2. Filter
3. Inverse FFT

Slide: Hoiem
Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?
Gaussian
Box Filter
Is convolution invertible?

• If convolution is just multiplication in the Fourier domain, isn’t deconvolution just division?
• Sometimes, it clearly is invertible (e.g. a convolution with an identity filter)
• In one case, it clearly isn’t invertible (e.g. convolution with an all zero filter)
• What about for common filters like a Gaussian?