Variations on the Hermann grid: an extinction illusion

Jacques Ninio  
Laboratoire de Physique Statistique\(^{(1)}\), École Normale Supérieure, 24 rue Lhomond, 75231 Paris cedex 05, France; e-mail: jacques.ninio@lps.ens.fr

Kent A Stevens  
Department of Computer Science, Deschutes Hall, University of Oregon, Eugene, OR 97403, USA; e-mail: kent@cs.uoregon.edu  
Received 21 September 1999, in revised form 21 June 2000

Abstract. When the white disks in a scintillating grid are reduced in size, and outlined in black, they tend to disappear. One sees only a few of them at a time, in clusters which move erratically on the page. Where they are not seen, the grey alleys seem to be continuous, generating grey crossings that are not actually present. Some black sparkling can be seen at those crossings where no disk is seen. The illusion also works in reverse contrast.

The Hermann grid (Brewster 1844; Hermann 1870) is a robust illusion. It is classically presented as a two-dimensional array of black squares, separated by rectilinear alleys. It is thought to be caused by processes of local brightness computation in arrays of
Course logistics

• My office hours have changed on Mondays (3 to 4, now) but are the same on Wednesdays (1 to 2).
• The second quiz has been moved up to Friday, December 2\textsuperscript{nd}.
Sparse to Dense Correspondence

Building Rome in a Day
By Sameer Agarwal, Yasutaka Furukawa, Noah Snavely, Ian Simon, Brian Curless, Steven M. Seitz, Richard Szeliski
Communications of the ACM, Vol. 54 No. 10, Pages 105-112
Correlation-based window matching

Textureless regions are non-distinct; high ambiguity for matches.
Stereo matching as energy minimization

\[ E = \alpha E_{\text{data}}(I_1, I_2, D) + \beta E_{\text{smooth}}(D) \]

\[ E_{\text{data}} = \sum_i (W_1(i) - W_2(i + D(i)))^2 \]

\[ E_{\text{smooth}} = \sum_{i, j \text{ neighbors}} \rho(D(i) - D(j)) \]

- Energy functions of this form can be minimized using graph cuts


Source: Steve Seitz
Better results…

Graph cut method


Ground truth

For the latest and greatest:  [http://www.middlebury.edu/stereo/](http://www.middlebury.edu/stereo/)

---

[Image of two compared images showing better results achieved with the Graph cut method compared to the ground truth, with a note on the Boykov et al. method and a link to Middlebury's stereo evaluation tool.]
Challenges

• Low-contrast; textureless image regions
• Occlusions
• Violations of brightness constancy (e.g., specular reflections)
• Really large baselines (foreshortening and appearance change)
• Camera calibration errors
Video

- A video is a sequence of frames captured over time
- Now our image data is a function of space \((x, y)\) and time \((t)\)
Motion estimation: Optical flow

*Optic flow* is the **apparent** motion of objects or surfaces

Will start by estimating motion of each pixel separately
Then will consider motion of entire image
Problem definition: optical flow

How to estimate pixel motion from image $I(x,y,t)$ to $I(x,y,t+1)$?

- Solve pixel correspondence problem
  - given a pixel in $I(x,y,t)$, look for nearby pixels of the same color in $I(x,y,t+1)$

Key assumptions

- **color constancy**: a point in $I(x,y)$ looks the same in $I(x,y,t+1)$
  - For grayscale images, this is brightness constancy
- **small motion**: points do not move very far

This is called the optical flow problem
Optical flow constraints (grayscale images)

Let’s look at these constraints more closely

- brightness constancy constraint (equation)
  \[ I(x, y, t) = I(x + u, y + v, t + 1) \]

- small motion: \((u \text{ and } v \text{ are less than } 1 \text{ pixel, or smooth})\)

  Taylor series expansion of \(I\):
  \[
  I(x + u, y + v) = I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \text{[higher order terms]}
  \]
  \[
  \approx I(x, y) + \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v
  \]}
Optical flow equation

- Combining these two equations

\[
0 = I(x + u, y + v, t + 1) - I(x, y, t)
\]
\[
\approx I(x, y, t + 1) + I_x u + I_y v - I(x, y, t)
\]

(Short hand: \( I_x = \frac{\partial I}{\partial x} \) for \( t \) or \( t+1 \))
Optical flow equation

- Combining these two equations

\[ 0 = I(x + u, y + v, t + 1) - I(x, y, t) \]
\[ \approx I(x, y, t + 1) + I_x u + I_y v - I(x, y, t) \]
\[ \approx [I(x, y, t + 1) - I(x, y, t)] + I_x u + I_y v \]
\[ \approx I_t + I_x u + I_y v \]
\[ \approx I_t + \nabla I \cdot \langle u, v \rangle \]

(Short hand: \( I_x = \frac{\partial I}{\partial x} \) for \( t \) or \( t+1 \))
Optical flow equation

- Combining these two equations

\[
0 = I(x+u, y+v, t+1) - I(x, y, t)
\approx I(x, y, t+1) + I_x u + I_y v - I(x, y, t)
\approx [I(x, y, t+1) - I(x, y, t)] + I_x u + I_y v
\approx I_t + I_x u + I_y v
\approx I_t + \nabla I \cdot <u, v>
\]

In the limit as \(u\) and \(v\) go to zero, this becomes exact

\[
0 = I_t + \nabla I \cdot <u, v>
\]

**Brightness constancy constraint equation**

\[
I_x u + I_y v + I_t = 0
\]
How does this make sense?

Brightness constancy constraint equation

\[ I_x u + I_y v + I_t = 0 \]

- What do the static image gradients have to do with motion estimation?
The brightness constancy constraint

Can we use this equation to recover image motion \((u,v)\) at each pixel?

\[ 0 = I_t + \nabla I \cdot <u,v> \quad \text{or} \quad I_x u + I_y v + I_t = 0 \]

- How many equations and unknowns per pixel?
  - One equation (this is a scalar equation!), two unknowns \((u,v)\)

The component of the motion perpendicular to the gradient (i.e., parallel to the edge) cannot be measured

If \((u,v)\) satisfies the equation, so does \((u+u', v+v')\) if

\[ \nabla I \cdot [u' \ v']^T = 0 \]
Aperture problem
Aperture problem
Aperture problem
The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion
The barber pole illusion

http://en.wikipedia.org/wiki/Barberpole_illusion
Solving the ambiguity...


- How to get more equations for a pixel?
- **Spatial coherence constraint**
  - Assume the pixel’s neighbors have the same (u,v)
  - If we use a 5x5 window, that gives us 25 equations per pixel

\[
0 = I_t(p_i) + \nabla I(p_i) \cdot [u \ v]
\]

\[
\begin{bmatrix}
I_x(p_1) & I_y(p_1) \\
I_x(p_2) & I_y(p_2) \\
\vdots & \vdots \\
I_x(p_{25}) & I_y(p_{25}) \\
\end{bmatrix}
\begin{bmatrix}
 u \\
v \\
\end{bmatrix}
= -
\begin{bmatrix}
I_t(p_1) \\
I_t(p_2) \\
\vdots \\
I_t(p_{25}) \\
\end{bmatrix}
\]
Solving the ambiguity...

- Least squares problem:

\[
\begin{bmatrix}
I_x(p_1) & I_y(p_1) \\
I_x(p_2) & I_y(p_2) \\
\vdots & \vdots \\
I_x(p_{25}) & I_y(p_{25})
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -
\begin{bmatrix}
I_t(p_1) \\
I_t(p_2) \\
\vdots \\
I_t(p_{25})
\end{bmatrix}
\]

\[
A \ d = b
\]

\[25\times 2 \ 2\times 1 \ 25\times 1\]
Matching patches across images

- Overconstrained linear system

\[
\begin{bmatrix}
I_x(p_1) & I_y(p_1) \\
I_x(p_2) & I_y(p_2) \\
\vdots & \vdots \\
I_x(p_{25}) & I_y(p_{25})
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -
\begin{bmatrix}
I_t(p_1) \\
I_t(p_2) \\
\vdots \\
I_t(p_{25})
\end{bmatrix}
\]

\[A \begin{bmatrix}
d
\end{bmatrix} = b\]

Least squares solution for \(d\) given by

\[
\begin{bmatrix}
\sum I_xI_x & \sum I_xI_y \\
\sum I_xI_y & \sum I_yI_y
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -
\begin{bmatrix}
\sum I_xI_t \\
\sum I_yI_t
\end{bmatrix}
\]

\[
A^T A
\]

\[
A^T b
\]

The summations are over all pixels in the \(K \times K\) window.
Conditions for solvability

Optimal \((u, v)\) satisfies Lucas-Kanade equation

\[
\begin{bmatrix}
\sum I_x I_x & \sum I_x I_y \\
\sum I_x I_y & \sum I_y I_y
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
= -
\begin{bmatrix}
\sum I_x I_t \\
\sum I_y I_t
\end{bmatrix}
\]

\(A^T A\)

\(A^T b\)

When is this solvable? I.e., what are good points to track?

- \(A^T A\) should be invertible
- \(A^T A\) should not be too small due to noise
  - eigenvalues \(\lambda_1\) and \(\lambda_2\) of \(A^T A\) should not be too small
- \(A^T A\) should be well-conditioned
  - \(\lambda_1 / \lambda_2\) should not be too large (\(\lambda_1 =\) larger eigenvalue)

Does this remind you of anything?

Criteria for Harris corner detector
Low texture region

\[ \sum \nabla I (\nabla I)^T \]
- gradients have small magnitude
- small \( \lambda_1 \), small \( \lambda_2 \)
Edge

\[ \sum \nabla I (\nabla I)^T \]
- large gradients, all the same
- large \( \lambda_1 \), small \( \lambda_2 \)
High textured region

\[ \sum \nabla I (\nabla I)^T \]

- gradients are different, large magnitudes
- large \( \lambda_1 \), large \( \lambda_2 \)
The aperture problem resolved
The aperture problem resolved

Perceived motion
Errors in Lucas-Kanade

• A point does not move like its neighbors
  • Motion segmentation

• Brightness constancy does not hold
  • Do exhaustive neighborhood search with normalized correlation - tracking features – maybe SIFT – more later….

• The motion is large (larger than a pixel)
  1. Not-linear: Iterative refinement
  2. Local minima: coarse-to-fine estimation
Revisiting the small motion assumption

• Is this motion small enough?
  • Probably not—it’s much larger than one pixel
  • How might we solve this problem?
Temporal aliasing causes ambiguities in optical flow because images can have many pixels with the same intensity. I.e., how do we know which ‘correspondence’ is correct?

To overcome aliasing: **coarse-to-fine estimation.**
Reduce the resolution!
Coarse-to-fine optical flow estimation

Gaussian pyramid of image 1

Gaussian pyramid of image 2

image 1

image 2

$u=1.25$ pixels

$u=2.5$ pixels

$u=5$ pixels

$u=10$ pixels
Coarse-to-fine optical flow estimation

Gaussian pyramid of image 1

run iterative L-K

warp & upsample

run iterative L-K

Gaussian pyramid of image 2

image 1

image 2
Optical Flow Results

Lucas-Kanade without pyramids

Fails in areas of large motion

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003
Optical Flow Results

Lucas-Kanade with Pyramids

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003
State-of-the-art optical flow, 2009

Start with something similar to Lucas-Kanade
+ gradient constancy
+ energy minimization with smoothing term
+ region matching
+ keypoint matching (long-range)

Large displacement optical flow, Brox et al., CVPR 2009
State-of-the-art optical flow, 2015

Deep convolutional network which accepts a pair of input frames and upsamples the estimated flow back to input resolution. Very fast because of deep network, near the state-of-the-art in terms of end-point-error.

State-of-the-art optical flow, 2015

Synthetic Training data

State-of-the-art optical flow, 2015

Results on Sintel

Optical flow

• Definition: optical flow is the *apparent* motion of brightness patterns in the image

• Ideally, optical flow would be the same as the motion field

• Have to be careful: apparent motion can be caused by lighting changes without any actual motion
  
  – Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination
Quiz 1 discussion