1) Explain precisely what the dot product measures. Then evaluate \langle1,2\rangle \cdot \langle3,4\rangle:

The dot product is a scalar. It measures the product of their lengths and of the cosine of the angle between them. Hence it is positive when that angle is less than 90°. It is zero when the two vectors are orthogonal to each other. \langle1,2\rangle \cdot \langle3,4\rangle = 3 + 8 = 11.

2) Compute \langle1,2\rangle .left. Explain precisely what it is:

\langle1,2\rangle .left = \langle-2,1\rangle. It is obtained by rotating \langle1,2\rangle ccw by 90°. Verify that the two vectors are orthogonal, since their dot-product is zero.

3) Evaluate \(V^2\), when \(V\) is the vector \langle3,4\rangle. Explain what \(V^2\) measures:

\(V^2 = 25\). It stands for \(V \cdot V\) and measures the square of the norm of \(V\).

4) Compute the result \(R\) of rotating point \(P=(2,3)\) by 30° around point \(Q=(3,5)\). First provide an exact formulation (using fractions, roots, sin, cos… if necessary) and then a numerical approximation:

\(QP=P-Q; c=\cos(30); s=\sin(30); I=<c,s>; J=I.left=<-s,c>; R=Q+(QP.x)I+(QP.y)J; R=(3,5)\leftarrow c,s\rightarrow 2\leftarrow -s,c>; R=(3.1339746,2.767949);\)

\(pt\ P = new\ pt(2,3); pt\ Q = new\ pt(3,5); pt\ R = P.makeRotatedBy(-PI*30/180,Q); // see page 2 for details\)

5) Let \((x_1,y_1)\) be the coordinates of point \(P\) in \([I_1,J_1,O_1]\). How would you compute its coordinates \((x_2,y_2)\) in \([I_2,J_2,O_2]\)? Provide a series of assignments/steps that compute \(x_2\) and \(y_2\) using operators (+, −, scaling, •…) on points and/or vectors. Provide a brief comment on what each step computes.

\(P=O_1+x_1I_1+y_1J_1; x_2=O_2P\cdot I_2; y_2=O_2P\cdot J_2;\)

6) Provide a valid expression for a point \(P\) located 1/3 along the way from \(A\) to \(B\) : \(A+AB/3\)

7) Vectors \(V\) and \(U\) are parallel when : \(V \cdot U .left = 0\), i.e. when \(V.xU.y=V.yU.x\)

8) A point \(R(t)\) starts at \(P\) and travels at constant velocity \(V\). Compute the time \(t\) when it hits the line passing through point \(Q\) and tangent to \(T\)? Include the derivation and explain briefly each step.

\(R(t)=P+tV; R(t)\ is\ on\ the\ line\ when\ QR(t)\cdot N=0,\ where\ N=T.left is\ the\ normal\ to\ the\ line.\ Substituting\ R(t) : (R(t)−Q)\cdot N=0: (P+tV−Q)\cdot N=0: (P−Q+tV)\cdot N=0: QP\cdot N+tV\cdot N=0:\ t=−(QP\cdot N)/(V\cdot N)\)

9) Assume that a disk\((C_1,r)\) with velocity \(V_1\) has just collided (i.e., is in tangential contact) with disk\((C_2,r)\) that has velocity \(V_2\). Explain how to compute their new velocities \(W_1\) and \(W_2\) after an elastic shock. (Assume both disks have the same mass.) Provide a formula or a series of assignments that evaluate \(W_1\) and \(W_2\) using operators (such as +, −, scaling, •) on points and/or vectors.

\(N=C_1C_1.unit; N_1=(V_1\cdot N)N; N_2=(V_2\cdot N)N; D= N_2−N_1; W_1=V_1+D; W_2=V_2−D;\)
You can use processing to test your solutions to geometric constructions. Here’s an example where I used Processing to test my solution to problem 4. The code in red is for visualization. Note that since the y-axis goes down in Processing, the coordinate system is inverted.

```java
pt P = new pt(200,300); fill(155,0,0); P.show(3); P.showLabel("P");
pt Q = new pt(300,500); fill(0,155,0); Q.show(3); Q.showLabel("Q");
vec QP = Q.makeVecTo(P); QP.showArrowAt(Q); mid(P,Q).showLabel("QP");
float c = cos(-PI*30/180), s = sin(-PI*30/180);
vec I = new vec(c,s); stroke(0,0,100); I.makeScaledBy(100).showArrowAt(Q);
vec J = I.left(); J.makeScaledBy(100).showArrowAt(Q);
pt R = Q.makeClone(); R.translateBy(QP.x,I); R.translateBy(QP.y,J);
fill(0,0,155); R.show(3); R.showLabel("R"); R.scaleBy(0.01); R.write();
```