1050 Quiz **QL04**, Jan 19, 2006. NAME:__________________________. Grade:_____ (____/10)
Closed book. Individual test. Do not look at other people's work. Please write legibly.

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1) To prove that $p \rightarrow q$, we can devise different types of proofs.

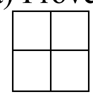
a) What is the difference between a direct proof and an indirect proof?

A direct proof shows that if p is true then q must be true; an indirect proof shows that if q is false then p must be false.

b) Is there such a thing as a "proof by counterexample"? Why or why not? (In other words, what is a counterexample used for?)

No, because a counterexample is used to <u>disprove</u> a proposition.

2) a) Prove that the following 2 x 2 grid can be covered using dominoes (of size 1 x 2).



(Simply cover this grid with two dominoes)

b) What type of proof was this? ***Proof by construction or existence proof***

3) Prove <u>by contradiction</u> that, for all integers n, if 3n + 2 is odd, then n is odd.

Assume that 3n + 2 is odd and n is even.
If n is even, then n = 2k for some integer k.
Then 3n + 2 = 3(2k) + 2 = 2(3k + 1) = 2m (where m = 3k + 1), which is even.
So 3n + 2 is even. But this contradicts the assumption that 3n + 2 is odd.
So the assumption must have been false.

4) Assume these three statements:

1.) Mike has a job or Leila has a job
2.) If Leila didn't go to the career fair, then Leila doesn't have a job
3.) Mike doesn't have a job or Leila is rich

Based on the above assumptions, do the conclusions below follow logically? Prove/disprove. (You may use English or propositions and logic operators, but be formal. When disproving a conclusion below, one way to do it is by pointing out a specific case that satisfies the above statements but causes the conclusion below to be false.)

a) If Leila has a job then Leila went to the career fair.

TRUE. This is the contrapositive of assumption 2 above, and the contrapositive of a true statement is true. i.e.: $p \rightarrow q \Rightarrow \neg q \rightarrow \neg p$

b) If Mike doesn't have a job, then Leila went to the career fair.

TRUE. If Mike doesn't have a job, then Leila must have a job,
by assumption 1 and because $(p \vee q) \wedge \neg p \Rightarrow q$ ***(a.k.a. disjunctive syllogism).***
If Leila has a job, then by assumption 2, Leila went to the career fair. (contrapositive of 2)

c) If Leila went to the career fair, then Mike doesn't have a job.

FALSE. Here is a counterexample: Suppose Leila went to the career fair, Leila doesn't have a job, Mike does have a job, and Leila is rich. This does not violate any of the above assumptions, but it does contradict the Mike not having a job, so it makes this statement false ($T \rightarrow F$ ***is false).***

d) Leila has a job or Leila is rich

TRUE. By assumptions 1 and 3, and by because $(p \vee q) \wedge (\neg p \vee r) \Rightarrow (q \vee r)$ ***a.k.a resolution.***
To put it differently:
Either Mike has a job or doesn't have a job.
If Mike doesn't have a job, then by assumption 1, Leila must have a job.
If Mike has a job then by assumption 3, Leila must be rich.
So either Leila has a job or Leila is rich.