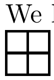
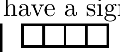
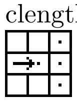
Notes: CS-7491 Lecture Notes

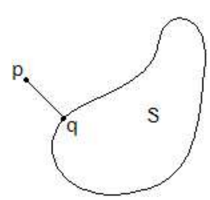
1/24/2006

1 Discussion on compressing a signature

We have a signature in a 64x256 pixel window. As a bitmap it takes up 2KB.
  One idea - break up the image into 4-pixel chunks and use Huffman encoding. File size after this was approximately 800 bytes, after compressing with zip, about 300-600 bytes.
Other idea - use run length encoding (such as gif). This resulted in approximately an 800 byte file.
Other ideas - treat as a sparse matrix?, treat as a curve parameterized by arclength?
 Jarek - we could treat it as a polygonal curve through the pixels. Assuming about 1.5 bits per pixel times 2K pixels covered, we get about 3000 bits.

2 Measures

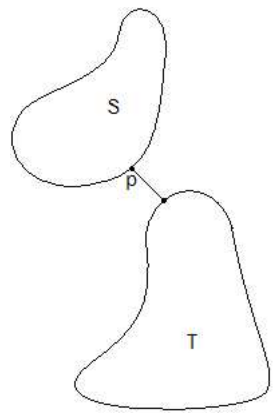
Suppose S defines a surface (or curve), and p and q represent points.

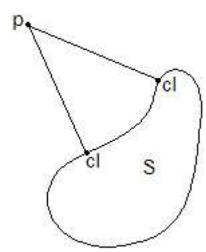


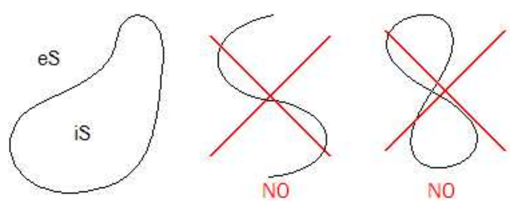
Distance $d(p,S) = \min_{q \in S} \|pq\|$.

$d(S,T) = \min_{p \in S} d(p,T)$.

Closest projection(s) of p on S: $cl(p,S) = \{q \in S : \|pq\| = d(p,S)\}$.







Assume S is a simple closed curve.
iS = interior region enclosed by S.
eS = exterior region enclosed by S.

signed distance $= -d(p, S)$ if $p \in iS$.

CUT (medial axis): $cut(S) = \{p : |cl(p, S)| > 1\}$ (i.e. the set of points with more than one closest projection.)

For $c \in cut(S)$, the radius $r_S(c)$ is the local thickness of the cut.

Given a cut and a radius one can reconstruct S. Take the union of open discs of radii $r_S(c)$ about each point c and this will be the complement of S.

Figure 1: CUT

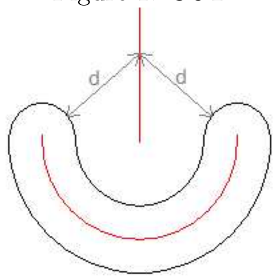
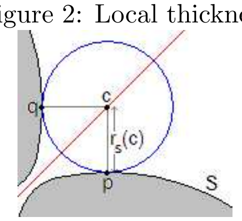
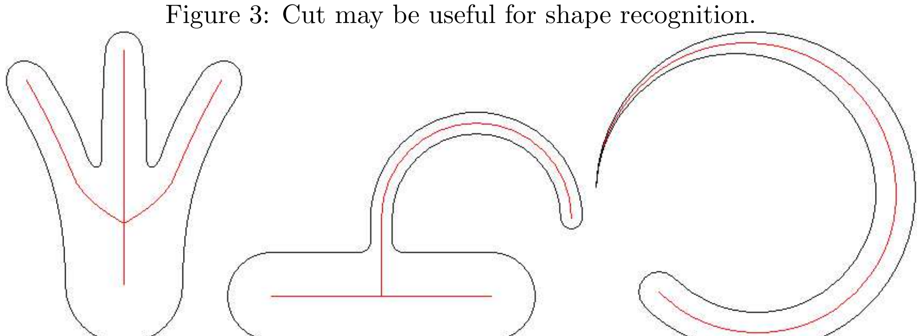


Figure 2: Local thickness



Definitions: interior cut = portion of $cut(S)$ within iS.
exterior cut = portion of $cut(S)$ within eS.

Figure 3: Cut may be useful for shape recognition.

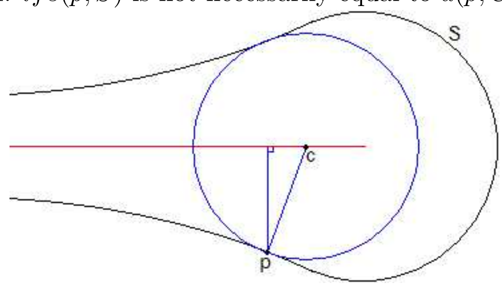


3 Feature size

Local feature size at $p \in S$: $lfs(p, S) = r$ = maximum radius of a ball with p on it's boundary but not any other point of S.
There are actually two lfs's:

ilfs - interior lfs
elfs - exterior lfs
lfs = min(ilfs, elfs)

Figure 4: $lfs(p, S)$ is not necessarily equal to $d(p, CUT(s))$!

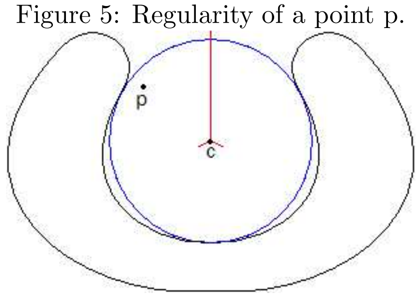


minimum feature size: $mfs(S) = d(S, cut(S)) = \min_{p \in S}(lfs(p, S))$.

Why does mfs matter? I can perturb S by less than mfs without risking a change in topology (subject to certain restrictions).

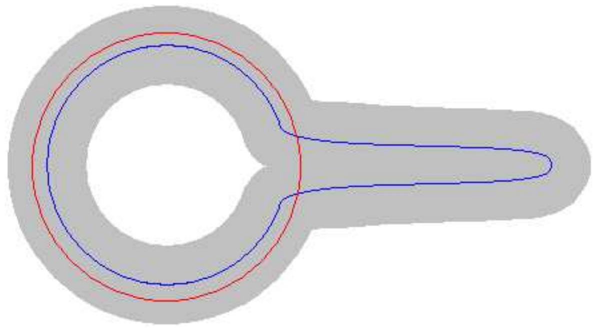
Regularity of p w.r.t. S: $Reg(p, S)$ = radius of the largest disc disjoint from S containing p. The center c of this disc is on the $cut(S)$.

Figure 5: Regularity of a point p.



Tolerance zone $z_r(S) = \{b : d(b, S) < r\}$
As r grows $z_r(S)$ may change topology. $z_r(S)$ has the "topology" of S if $r < mfs(S)$.

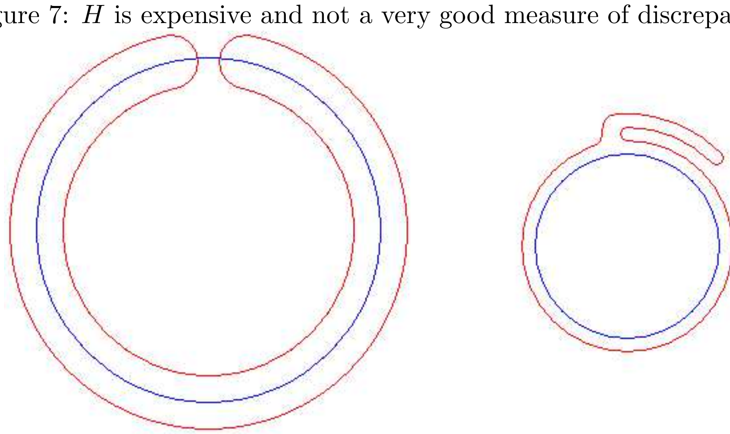
Figure 6: Tolerance zone could still allow red curve to deviate a lot from blue curve.



4 Hausdorff distance:

$H(S, T) = \min r$ s.t. $S \subset z_r(T)$ and $T \subset z_r(S)$

Figure 7: H is expensive and not a very good measure of discrepancy.

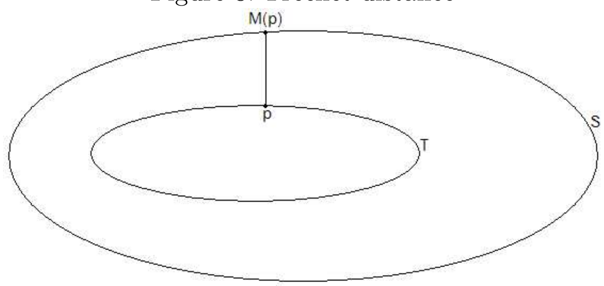


Ideally, we want a bijection between S and T (one-to-one and onto).

5 Frechét distance

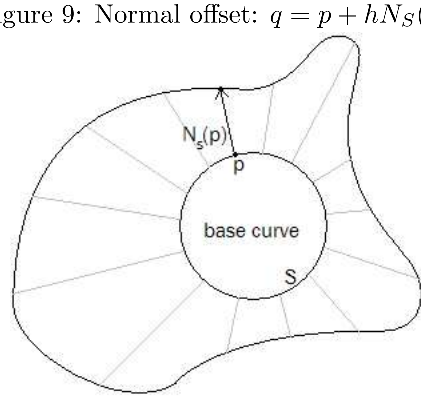
$F(S, T) = \min_{M:S \to T} \max_{p \in S} \|p - M(p)\|$

Figure 8: Frechét distance



6 Normal offset

Figure 9: Normal offset: $q = p + hN_S(p)$.



S & T are "normal compatible" if each one is a normal offset of the other.

Sufficient condition for normal compatibility:
$H(A, B) < (2 - \sqrt{2})\min(mfs(A), mfs(B))$

Figure 10: Normal mapping may not be symmetric.

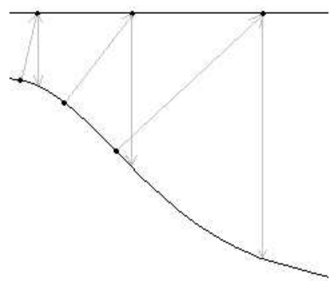


Figure 11: Essentially, we don't want to ever see the other curve "going backwards".

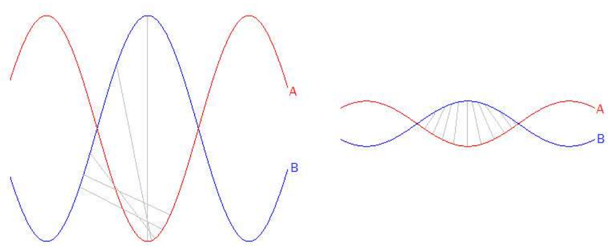
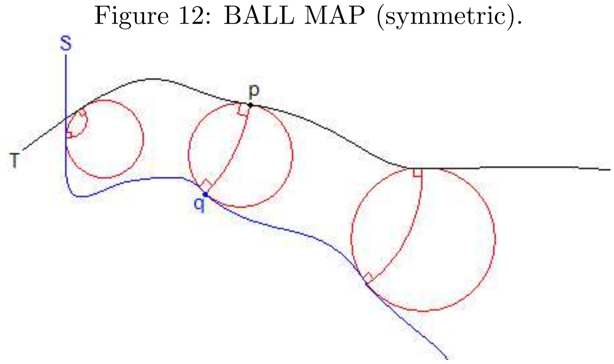


Figure 12: BALL MAP (symmetric).



- ball map minimizes "stretching" between parameterizations.