



# Join Algorithms

CREATING THE NEXT®

# Administrivia

---

- Sheet 4 and assignment 4 due on Nov 17
- Extra credit assignment due on Dec 01

# Today's Agenda

---

Recap

Overview

Nested Loop Join

Sort-Merge Join

Hash Join

Conclusion

# Recap

# External Merge Sort

---

- Divide-and-conquer sorting algorithm that splits the data set into separate **runs** and then sorts them individually.
- **Phase 1 – Sorting**
  - ▶ Sort blocks of data that fit in main-memory and then write back the sorted blocks to a file on disk.
- **Phase 2 – Merging**
  - ▶ Combine sorted sub-files into a single larger file.

# Aggregation

---

- Collapse multiple tuples into a single scalar value.
- Two implementation choices:
  - ▶ Sorting
  - ▶ Hashing

# Hashing Aggregate

---

- Populate an **ephemeral hash table** as the DBMS scans the table.
- For each record, check whether there is already an entry in the hash table:
  - ▶ GROUP BY: Perform aggregate computation.
  - ▶ DISTINCT: Discard duplicates.
- If everything fits in memory, then it is easy.
- If the DBMS must spill data to disk, then we need to be smarter.

# Overview



## Why do we need to join?

---

- We normalize tables in a relational database to avoid unnecessary repetition of information.
- We use the join operator to reconstruct the original tuples without any information loss.

## Denormalized Tables

---

**Artists** (ID, Artist, Year, City)

**Albums** (ID, Album, Artist, Year)

	<u>ID</u>	Artist	Year	City
<b>Artists</b>	1	Mozart	1756	Salzburg
	2	Beethoven	1770	Bonn

	<u>ID</u>	Album	Artist	Year
<b>Albums</b>	1	The Marriage of Figaro	Mozart	1786
	2	Requiem Mass In D minor	Mozart	1791
	3	Für Elise	Beethoven	1867

# Normalized Tables

---

**Artists** (ID, Artist, Year, City)

**Albums** (ID, Album, Year)

**ArtistAlbum** (Artist\_ID, Album\_ID)

	<u>Artist_ID</u>	<u>Album_ID</u>
<b>ArtistAlbum</b>	1	1
	2	1
	2	2

# Join Algorithms

---

- We will focus on combining **two tables** at a time with **inner equi-join** algorithms.
  - ▶ These algorithms can be tweaked to support other types of joins.
- In general, we want the smaller table to always be the left table (**outer table**) in the query plan.

# Join Operators

- Decision 1: Output

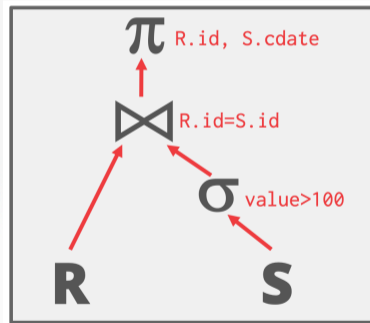
- ▶ What data does the join operator emit to its parent operator in the query plan tree?

- 

- Decision 2: Cost Analysis Criteria

- ▶ How do we determine whether one join algorithm is better than another?

```
SELECT R.id, S.cdate  
FROM R JOIN S  
ON R.id = S.id  
WHERE S.value > 100
```



# Join Operator Output

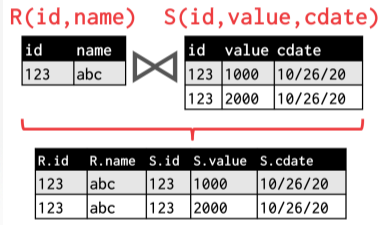
---

- For a tuple  $r \in R$  and a tuple  $s \in S$  that match on join attributes, concatenate  $r$  and  $s$  together into a new tuple.
- Contents can vary:
  - ▶ Depends on query processing model
  - ▶ Depends on storage model
  - ▶ Depends on the query

## Join Operator Output: Data

---

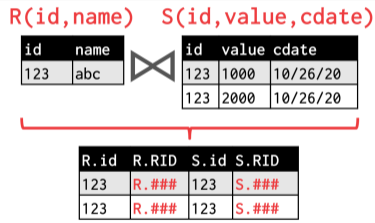
- Copy the values for the attributes in outer and inner tuples into a new output tuple.
- Subsequent operators in the query plan never need to go back to the base tables to get more data.



## Join Operator Output: Record Ids

---

- Only copy the joins keys along with the record ids of the matching tuples.
- Ideal for **column stores** because the DBMS does not copy data that is not need for the query.
- This is called **late materialization**.





# I/O Cost Analysis

---

- Assume:
  - ▶ M pages in table R, m tuples in R
  - ▶ N pages in table S, n tuples in S
- Cost Metric: Number of IO operations to compute join
- We will ignore output costs (since that depends on the data and we cannot compute that yet).

```
SELECT R.id, S.cdate  
FROM R JOIN S  
ON R.id = S.id  
WHERE S.value > 100
```

## Join vs Cross-Product

---

- $R \bowtie S$  is the most common operation and thus must be carefully optimized.
- $R \times S$  followed by a selection is inefficient because the cross-product is large.
- There are many algorithms for reducing join cost, but no algorithm works well in all scenarios.

# Join Algorithms

---

- Nested Loop Join
  - ▶ Naïve
  - ▶ Block
  - ▶ Index
- Sort-Merge Join
- Hash Join

# Nested Loop Join

# Nested Loop Join

---

**R** (id, name)

**S** (id, value, cdate)

**operator** NestedLoopJoin( $R, S$ ):  
  for each tuple  $r \in R$ : // Outer Table  
    for each tuple  $s \in S$ : // Inner Table  
      emit, if  $r$  and  $s$  match

# Naïve Nested Loop Join

---

- Why is this algorithm naïve?
  - ▶ For every tuple in **R**, it scans **S** once
- R: M pages, m tuples
- S: N pages, n tuples
- **Cost**:  $M + (m \times N)$

# Naïve Nested Loop Join

---

- Example Database:
  - ▶ Table R:  $M = 1000$  pages,  $m = 100,000$  tuples
  - ▶ Table S:  $N = 500$  pages,  $n = 40,000$  tuples
  - ▶ Each page = 4 KB  $\implies$  Database size = 6 MB
- **Cost Analysis:**
  - ▶  $M + (m \times N) = 1000 + (100000 \times 500) = 50,001,000$  IOs
  - ▶ At 0.1 ms/IO, Total time  $\approx$  1.3 hours
- What if smaller table (S) is used as the outer table?
  - ▶  $N + (n \times M) = 500 + (40000 \times 1000) = 40,000,500$  IOs
  - ▶ At 0.1 ms/IO, Total time  $\approx$  1.1 hours

# Block Nested Loop Join

---

**R** (id, name)

**S** (id, value, cdate)

**operator** BlockNestedLoopJoin( $R, S$ ):

for each block  $b_R \in R$ : // Outer Table

for each block  $b_S \in S$ : // Inner Table

for each tuple  $r \in b_R$ :

for each tuple  $s \in b_S$ :

emit, if  $r$  and  $s$  match



# Block Nested Loop Join

---

- This algorithm performs fewer disk accesses.
  - ▶ For every block in R, it scans S once
- **Cost:**  $M + (M \times N)$

# Block Nested Loop Join

---

- Which one should be the outer table?
  - ▶ The smaller table in terms of number of pages

# Block Nested Loop Join

---

- Example Database:
  - ▶ Table R:  $M = 1000$  pages,  $m = 100,000$  tuples
  - ▶ Table S:  $N = 500$  pages,  $n = 40,000$  tuples
- **Cost Analysis:**
  - ▶  $M + (M \times N) = 1000 + (1000 \times 500) = 501,000$  IOs
  - ▶ At 0.1 ms/IO, Total time  $\approx 50$  seconds

## External Block Nested Loop Join

---

- What if we have **B** buffers available?
  - ▶ Use **B-2** buffers for scanning the outer table.
  - ▶ Use one buffer for the inner table, one buffer for storing output.

## External Block Nested Loop Join

---

**R** (id, name)

**S** (id, value, cdate)

**operator** ExternalBlockNestedLoopJoin( $R, S$ ):

for each B-2 block  $b_R \in R$ : // Outer Table

for each block  $b_S \in S$ : // Inner Table

for each tuple  $r \in b_R$ :

for each tuple  $s \in b_S$ :

emit, if  $r$  and  $s$  match

## Block Nested Loop Join

---

- This algorithm uses  $B-2$  buffers for scanning  $R$ .
- **Cost:**  $M + (\lceil M / (B-2) \rceil \times N)$
- What if the outer relation completely fits in memory (*i.e.*,  $B-2 > M$ )?
  - ▶ **Cost:**  $M + N = 1000 + 500 = 1500$  IOs
  - ▶ At 0.1 ms/IO, Total time  $\approx 0.15$  seconds

# Nested Loop Join

---

- Why do basic nested loop joins suck?
  - ▶ For each tuple in the outer table, we must do a sequential scan to check for a match in the inner table.
- We can avoid sequential scans by using an index to find inner table matches.
  - ▶ Use an existing index for the join.
  - ▶ Or build an index on the fly (*e.g.*, hash table, B+Tree).

# Index Nested Loop Join

---

**R** (id, name)

**S** (id, value, cdate)

**Index on S** (id)

**operator** IndexNestedLoopJoin( $R, S$ ):

for each tuple  $r \in R$ : // Outer Table

for each tuple  $s \in \text{Index}(r_i = s_i)$ : // Index on Inner Table

emit, if  $r$  and  $s$  match



# Index Nested Loop Join

---

- Assume the cost of each index probe is some constant  $C$  per tuple.
- Cost:  $M + (m \times C)$

## Summary

---

- Pick the smaller table as the outer table.
- Buffer as much of the outer table in memory as possible.
- Loop over the inner table or use an index if available.

# Sort-Merge Join

# Sort-Merge Join

---

- **Phase 1: Sort**
  - ▶ Sort both tables on the join key(s).
- **Phase 2: Merge**
  - ▶ We can then use the external merge sort algorithm to join the sorted tables.
  - ▶ Step through the two sorted tables with cursors and emit matching tuples.
  - ▶ May need to backtrack depending on the join type.

# Sort-Merge Join

---

**R** (id, name)

**S** (id, value, cdate)

**operator** SortMergeJoin( $R, S$ ):

sort  $R, S$  on join keys

$cursor_R \leftarrow R_{sorted}, cursor_S \leftarrow S_{sorted}$

while  $cursor_R$  and  $cursor_S$ :

if  $cursor_R > cursor_S$ :

increment  $cursor_S$

else if  $cursor_R < cursor_S$ :

increment  $cursor_R$

else if  $cursor_R$  and  $cursor_S$  match:

emit

increment  $cursor_S$

# Sort-Merge Join

---

R(id, name)

id	name
600	Mark
200	Rahul
100	Maria
300	Li
500	Shiyi
700	Alex
200	Peter
400	Ranveer

S(id, value, cdate)

id	value	cdate
100	2222	10/27/20
500	7777	10/27/20
400	6666	10/27/20
100	9999	10/27/20
200	8888	10/27/20

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

# Sort-Merge Join

---

R(id, name)

id	name
600	Mark
200	Rahul
100	Maria
300	Li
500	Shiyi
700	Alex
200	Peter
400	Ranveer

↑  
*Sort!*

S(id, value, cdate)

id	value	cdate
100	2222	10/27/20
500	7777	10/27/20
400	6666	10/27/20
100	9999	10/27/20
200	8888	10/27/20

↑  
*Sort!*

```
SELECT R.id, S.cdate
FROM R JOIN S
      ON R.id = S.id
WHERE S.value > 100
```

# Sort-Merge Join

R(id, name)

id	name
100	Maria
200	Rahul
200	Peter
300	Li
400	Ranveer
500	Shiyi
600	Mark
700	Alex

↑  
*Sort!*

S(id, value, cdate)

id	value	cdate
100	2222	10/27/20
100	9999	10/27/20
200	8888	10/27/20
400	6666	10/27/20
500	7777	10/27/20

↑  
*Sort!*


```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```



# Sort-Merge Join


---

R(id, name)



id	name
100	Maria
200	Rahul
200	Peter
300	Li
400	Ranveer
500	Shiyi
600	Mark
700	Alex

S(id, value, cdate)



id	value	cdate
100	2222	10/27/20
100	9999	10/27/20
200	8888	10/27/20
400	6666	10/27/20
500	7777	10/27/20

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Maria	100	2222	10/27/20

# Sort-Merge Join

**R(id,name)**

id	name
100	Maria
200	Rahul
200	Peter
300	Li
400	Ranveer
500	Shiyi
600	Mark
700	Alex

**S(id,value,cdate)**

id	value	cdate
100	2222	10/27/20
100	9999	10/27/20
200	8888	10/27/20
400	6666	10/27/20
500	7777	10/27/20

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

**Output Buffer**

R.id	R.name	S.id	S.value	S.cdate
100	Maria	100	2222	10/27/20
100	Maria	100	9999	10/27/20

# Sort-Merge Join

R(id, name)

id	name
100	Maria
200	Rahul
200	Peter
300	Li
400	Ranveer
500	Shiyi
600	Mark
700	Alex

S(id, value, cdate)

id	value	cdate
100	2222	10/27/20
100	9999	10/27/20
200	8888	10/27/20
400	6666	10/27/20
500	7777	10/27/20

```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Maria	100	2222	10/27/20
100	Maria	100	9999	10/27/20

# Sort-Merge Join

R(id, name)

id	name
100	Maria
200	Rahul
200	Peter
300	Li
400	Ranveer
500	Shiyi
600	Mark
700	Alex



S(id, value, cdate)

id	value	cdate
100	2222	10/27/20
100	9999	10/27/20
200	8888	10/27/20
400	6666	10/27/20
500	7777	10/27/20



```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Maria	100	2222	10/27/20
100	Maria	100	9999	10/27/20

# Sort-Merge Join

R(id,name)

id	name
100	Maria
200	Rahul
200	Peter
300	Li
400	Ranveer
500	Shiyi
600	Mark
700	Alex



S(id,value,cdate)

id	value	cdate
100	2222	10/27/20
100	9999	10/27/20
200	8888	10/27/20
400	6666	10/27/20
500	7777	10/27/20



```
SELECT R.id, S.cdate
FROM R JOIN S
ON R.id = S.id
WHERE S.value > 100
```

Output Buffer

R.id	R.name	S.id	S.value	S.cdate
100	Maria	100	2222	10/27/20
100	Maria	100	9999	10/27/20
200	Peter	200	8888	10/27/20
200	Peter	200	8888	10/27/20
400	Ranveer	200	6666	10/27/20
500	Shiyi	500	7777	10/27/20

# Sort-Merge Join

---

- Sort Cost (**R**):  $2M \times (1 + \lceil \log_{B-1} \lceil M / B \rceil \rceil)$
- Sort Cost (**S**):  $2N \times (1 + \lceil \log_{B-1} \lceil N / B \rceil \rceil)$
- Merge Cost:  $(M + N)$
- **Total Cost:** Sort + Merge

## Sort-Merge Join

---

- Example Database:
  - ▶ Table R:  $M = 1000$  pages,  $m = 100,000$  tuples
  - ▶ Table S:  $N = 500$  pages,  $n = 40,000$  tuples
- With  $B=100$  buffer pages, both R and S can be sorted in two passes:
  - ▶ Sort Cost (R) =  $2000 \times (1 + \lceil \log_{99} 1000 / 100 \rceil) = 4000$  IOs
  - ▶ Sort Cost (S) =  $1000 \times (1 + \lceil \log_{99} 500 / 100 \rceil) = 2000$  IOs
  - ▶ Merge Cost =  $(1000 + 500) = 1500$  IOs
  - ▶ Total Cost =  $4000 + 2000 + 1500 = 7500$  IOs
  - ▶ At 0.1 ms/IO, Total time  $\approx 0.75$  seconds

## Sort-Merge Join

---

- The worst case for the merging phase is when the join attribute of all of the tuples in both relations contain the same value.
- Cost:  $(M \times N) + (\text{sort cost})$



## When is Sort-Merge Join Useful?

---

- One or both tables are already sorted on join key.
- Output must be sorted on join key.
- The input relations may be sorted by either by an explicit sort operator, or by scanning the relation using an index on the join key.



# Hash Join

---

- If tuple  $r \in R$  and a tuple  $s \in S$  satisfy the join condition, then they have the same value for the join attributes.
- If that value is hashed to some partition  $i$ , the R tuple must be in  $r_i$  and the S tuple in  $s_i$ .
- Therefore, R tuples in  $r_i$  need only to be compared with S tuples in  $s_i$ .

# Basic Hash Join Algorithm

---

- **Phase 1: Build**
  - ▶ Scan the outer table and populate a hash table using the hash function  $h_1$  on the join attributes.
- **Phase 2: Probe**
  - ▶ Scan the inner table and use  $h_1$  on each tuple to jump to a location in the hash table and find a matching tuple.

# Basic Hash Join Algorithm

---

**R** (id, name)

**S** (id, value, cdate)

**operator** BasicHashJoin( $R, S$ ):

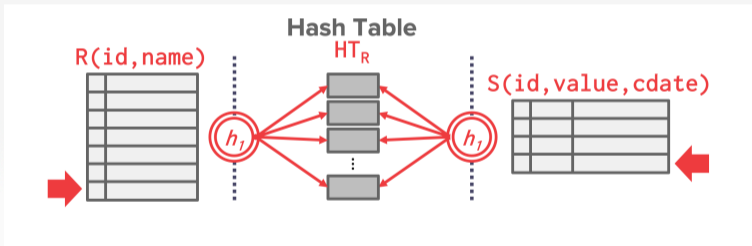
build hash table  $HT_R$  for  $R$

for each tuple  $s \in S$

emit, if  $h_1(s)$  in  $HT_R$

# Basic Hash Join Algorithm

---



# Hash Table Contents

---

- **Key**: The attribute(s) that the query is joining the tables on.
- **Value**: Depends on what the parent operator above the join in the query plan expects as its input.
  - ▶ **Approach 1: Full Tuple**
    - ★ Avoid having to retrieve the outer table's tuple contents on a match.
    - ★ Takes up more space in memory.
  - ▶ **Approach 2: Tuple Identifier**
    - ★ Ideal for column stores because the DBMS does **not** fetch data from disk unless needed.
    - ★ Also better if join selectivity is low.

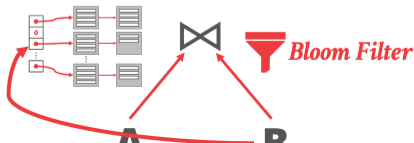
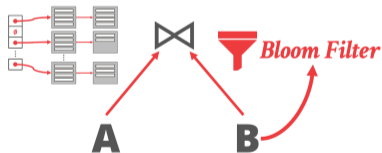
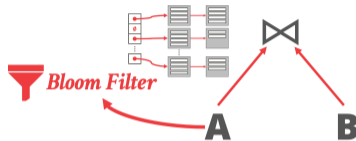
# Probe Phase Optimization

---

- Create a **bloom filter** during the build phase when the key is likely to **not** exist in the hash table.
  - ▶ Threads check the filter before probing the hash table.
  - ▶ This will be faster since the filter will fit in CPU caches.
  - ▶ *a.k.a.*, sideways information passing.



# Probe Phase Optimization



# Hash Join

---

- What happens if we do not have enough memory to fit the entire hash table?
- We do **not** want to let the buffer pool manager swap out the hash table pages randomly.

# Grace Hash Join

---

- Hash join when tables do **not** fit in memory.
  - ▶ **Build Phase:** Hash both tables on the join attribute into partitions.
  - ▶ **Probe Phase:** Compares tuples in corresponding partitions for each table.
- Named after the **GRACE database machine** from Japan in the 1980s.



GRACE  
*University of Tokyo*

## Grace Hash Join

---

- Hash R into  $(0, 1, \dots, max)$  buckets.
- Hash S into the same number of buckets with the same hash function.
- Join each pair of matching buckets between R and S.

# Grace Hash Join

---

**R** (id, name)

**S** (id, value, cdate)

**operator** Grace Hash Join( $R, S$ ):

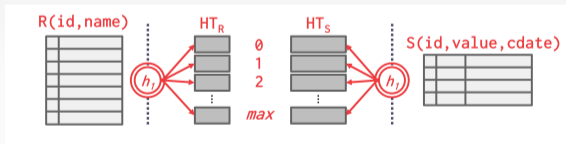
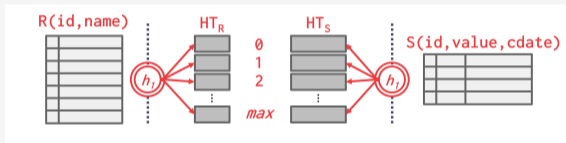
for bucket  $i \in [0, max]$

for each tuple  $r \in \text{bucket } R_i$

for each tuple  $s \in \text{bucket } S_i$

emit, if  $r$  and  $s$  match

# Grace Hash Join



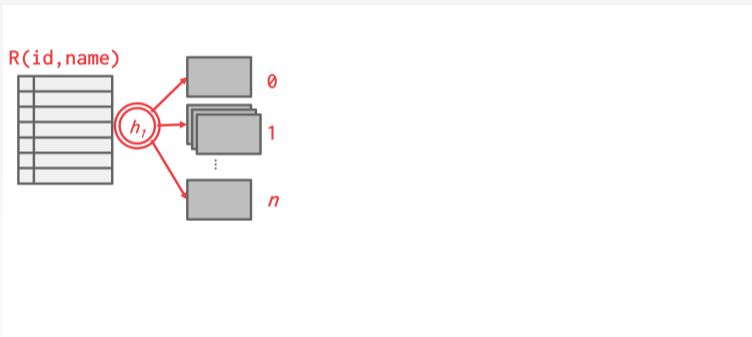
# Grace Hash Join

---

- If the buckets do not fit in memory, then use recursive partitioning to split the tables into chunks that will fit.
  - ▶ Build another hash table for  $bucket_{R_i}$  using hash function  $h_2$  (with  $h_2 \neq h_1$ ).
  - ▶ Then probe it for each tuple of the other table's bucket at that level.

# Recursive Partitioning

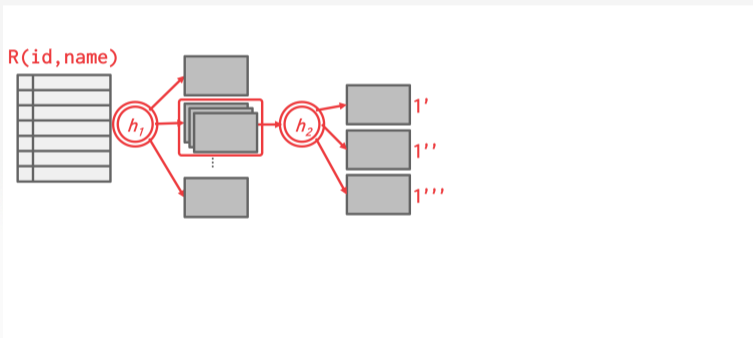
---





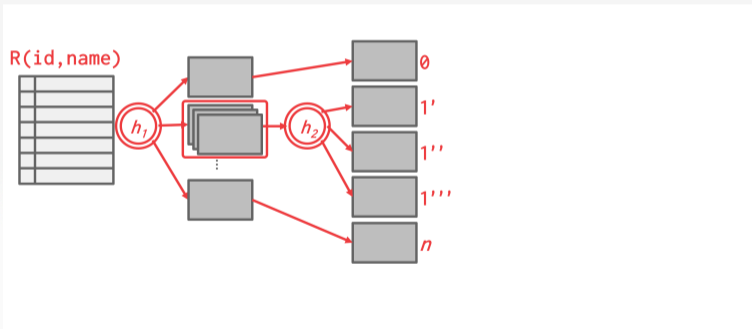
# Recursive Partitioning

---

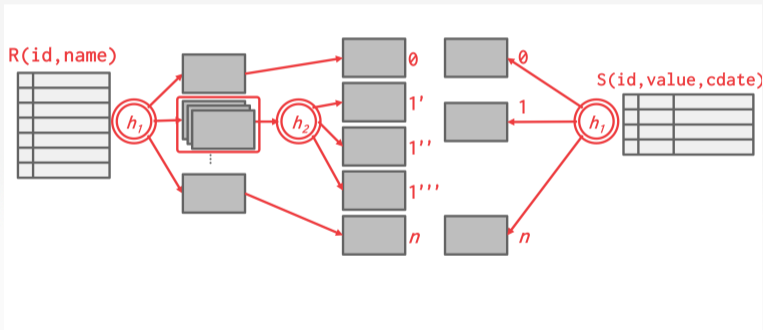


# Recursive Partitioning

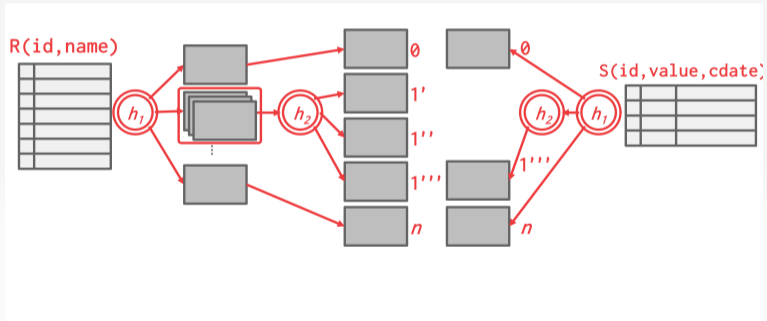
---



# Recursive Partitioning



# Recursive Partitioning



# Grace Hash Join

---

- **Partitioning Phase:**
  - ▶ Read+Write both tables
  - ▶  $2 \times (M + N)$  IOs
- **Probing Phase:**
  - ▶ Read both tables
  - ▶  $M + N$  IOs
- **Total Cost:**  $3 \times (M + N)$

# Grace Hash Join

---

- Example Database:
  - ▶ Table R:  $M = 1000$  pages,  $m = 100,000$  tuples
  - ▶ Table S:  $N = 500$  pages,  $n = 40,000$  tuples
- Cost Analysis:
  - ▶  $3 \times (M + N) = 3 \times (1000 + 500) = 4,500$  IOs
  - ▶ At 0.1 ms/IO, Total time  $\approx 0.45$  seconds

## Observation

---

- If the DBMS knows the size of the outer table, then it can use a **static hash table**.
  - ▶ Less computational overhead for build / probe operations.
- If we do not know the size, then we have to use a **dynamic hash table** or allow for overflow pages.

# Conclusion



## Join Algorithms: Summary

---

Join Algorithm	IO Cost	Example
Simple Nested Loop Join	$M + (m \times N)$	1.3 hours
Block Nested Loop Join	$M + (M \times N)$	50 seconds
Index Nested Loop Join	$M + (M \times C)$	Variable
Sort-Merge Join	$M + N + (\text{sort cost})$	0.75 seconds
Hash Join	$3 \times (M + N)$	0.45 seconds

# Conclusion

---

- Hashing is almost always better than sorting for operator execution.
- Caveats:
  - ▶ Sorting is better on non-uniform data.
  - ▶ Sorting is better when result needs to be sorted.
- Good DBMSs use either or both.
- Next Class
  - ▶ Composing operators together to execute queries.