



DATA ANALYTICS USING DEEP LEARNING GT 8803 // FALL 2019 // JOY ARULRAJ

LECTURE #19: DEEP REINFORCEMENT LEARNING

CREATING THE NEXT®

ADMINISTRIVIA

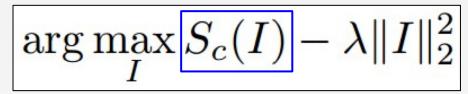
Reminders

- Mid-term grades have been released
- Code reviews due on Wednesday
- Project checkpoint #2 postponed to Monday

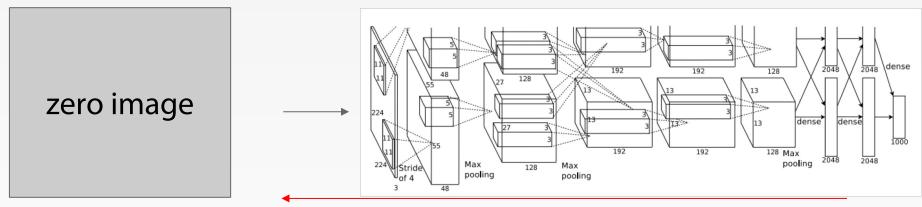


LAST TIME: VISUALIZING CNN FEATURES — GRADIENT ASCENT

1. Initialize image to zeros



score for class c (before Softmax)



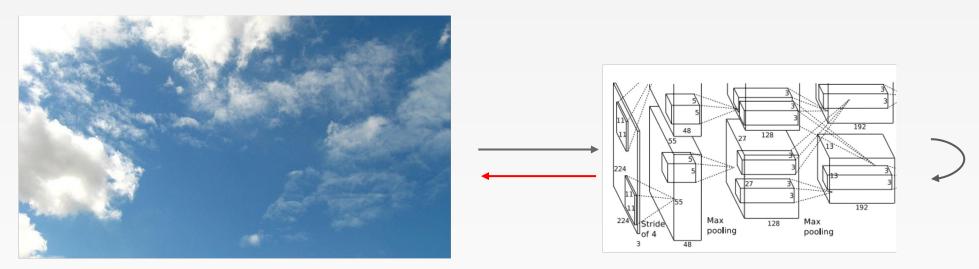
Repeat:

- 2. Forward image to compute current scores
- 3. Backprop to get gradient of neuron value with respect to image pixels
- 4. Make a small update to the image



LAST TIME: DEEPDREAM — AMPLIFY EXISTING FEATURES

Rather than synthesizing an image to maximize a specific neuron, instead try to **amplify** the neuron activations at some layer in the network



Choose an image and a layer in a CNN; repeat:

- 1. Forward: compute activations at chosen layer
- 2. Set gradient of chosen layer equal to its activation *
- Backward: Compute gradient on image
- 4. Update image

Equivalent to:

$$I^* = arg max_I \sum_i f_i(I)^2$$

Mordvintsev, Olah, and Tyka, "Inceptionism: Going Deeper into Neural Networks", <u>Google Research Blog</u>. Images are licensed under <u>CC-BY 4.0</u>

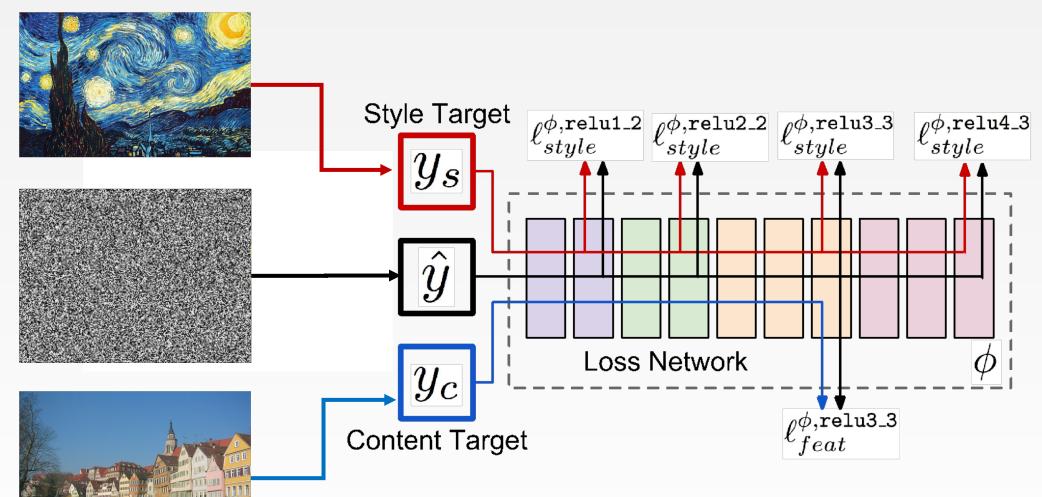


LAST TIME: NEURAL STYLE TRANSFER



Output image (Start with noise)

Content image



Gatys, Ecker, and Bethge, "Image style transfer using convolutional neural networks", CVPR 2016 Figure adapted from Johnson, Alahi, and Fei-Fei, "Perceptual Losses for Real-Time Style Transfer and Super-Resolution", ECCV 2016. Copyright Springer, 2016. Reproduced for educational purposes.

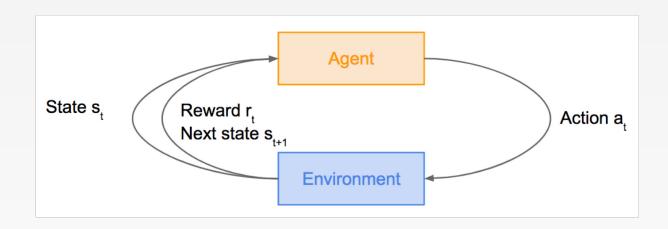


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TODAY: REINFORCEMENT LEARNING

Problems involving an **agent** interacting with an **environment**, which provides numeric **reward** signals

Goal: Learn how to take actions in order to maximize reward





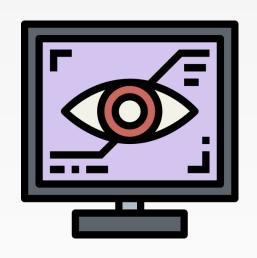
Atari games figure copyright Volodymyr Mnih et al., 2013. Reproduced with permission.



OVERVIEW

- What is Reinforcement Learning?
- Markov Decision Processes
- RL Algorithms
 - Q-Learning
 - Policy Gradients



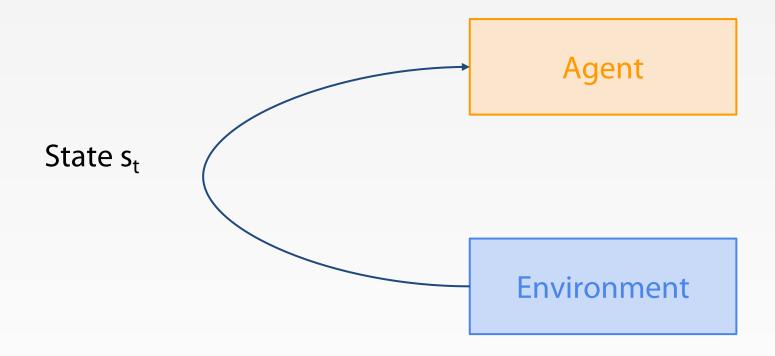




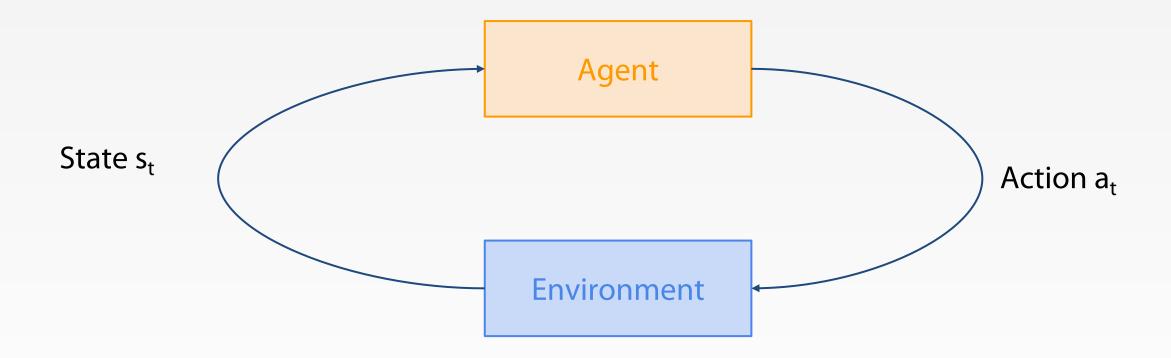
Agent

Environment

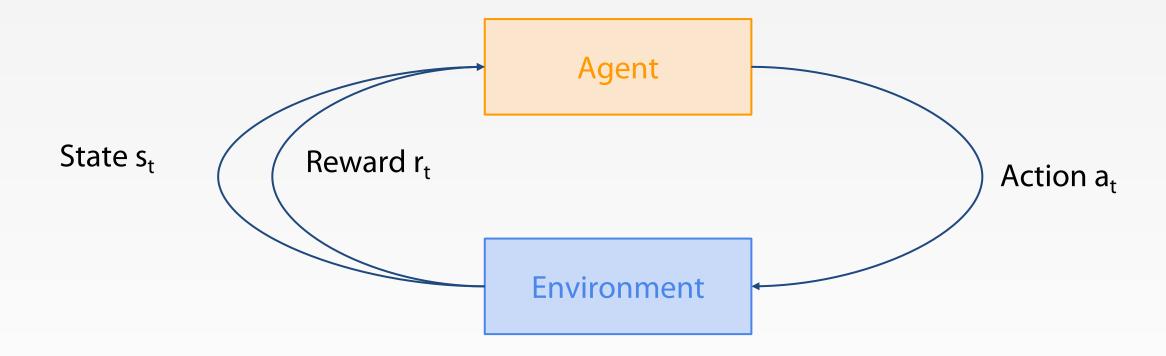




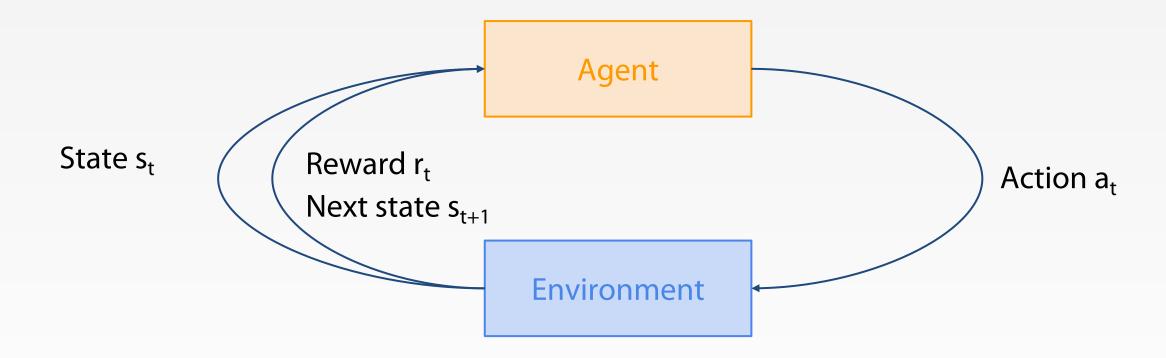






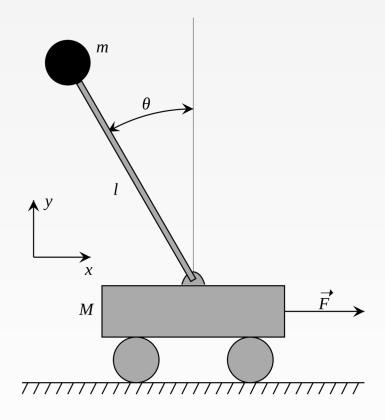








CART-POLE PROBLEM



Objective: Balance a pole on top of a movable cart

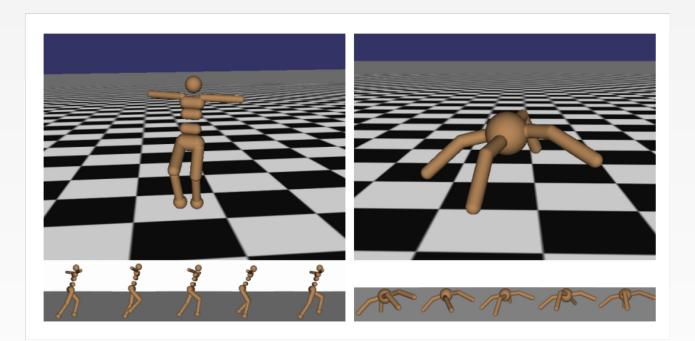
State: angle, angular speed, position, horizontal velocity

Action: horizontal force applied on the cart

Reward: 1 at each time step if the pole is upright



ROBOT LOCOMOTION



Objective: Make the robot move forward

State: Angle and position of the joints

Action: Torques applied on joints

Reward: 1 at each time step upright +

forward movement

Figures copyright John Schulman et al., 2016. Reproduced with permission.



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ATARI GAMES



Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step

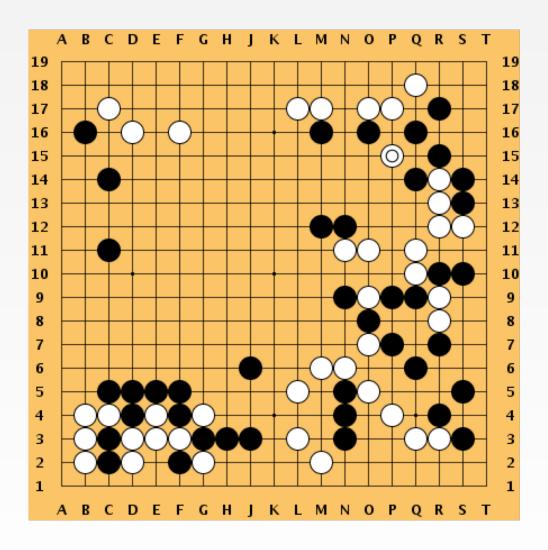
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GO



Objective: Win the game!

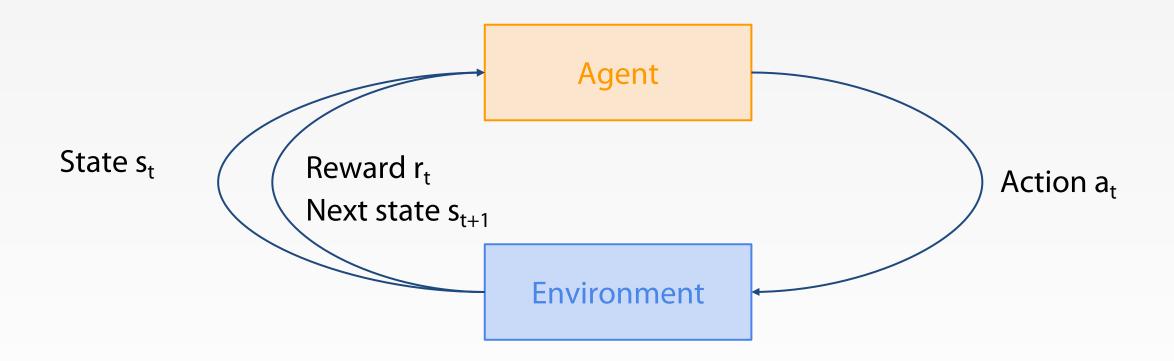
State: Position of all pieces

Action: Where to put the next piece down

Reward: 1 if win at the end of the game, 0 otherwise



HOW CAN WE MATHEMATICALLY FORMALIZE THE RL PROBLEM?







MARKOV DECISION PROCESS



MARKOV DECISION PROCESS

- Mathematical formulation of the RL problem
- Markov property: Current state completely characterizes the state of the world

Defined by: $(\mathcal{S},\mathcal{A},\mathcal{R},\mathbb{P},\gamma)$

 \mathcal{S} : set of possible states

 \mathcal{A} : set of possible actions

 $\mathcal{R}\,$: distribution of reward given (state, action) pair

p: transition probability i.e. distribution over next state given (state, action) pair

 γ : discount factor



MARKOV DECISION PROCESS

- At time step t=0, environment samples initial state $s_0 \sim p(s_0)$
- Then, for t=0 until done:
 - Agent selects action a_t
 - Environment samples reward $r_t \sim R(.|s_t, a_t)$
 - Environment samples next state $s_{t+1} \sim P(.|s_t, a_t)$
 - Agent receives reward r_t and next state s_{t+1}
- A policy π is a function from S to A that specifies what action to take in each state
- **Objective**: find policy π^* that maximizes cumulative discounted reward: $\sum_{t\geq 0} \gamma^t r_t$



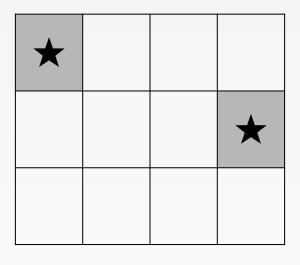
A SIMPLE MDP: GRID WORLD

actions = {

- 1. Right →
- 2. Left •••
- з. **Up**
- 4. Down

}

states

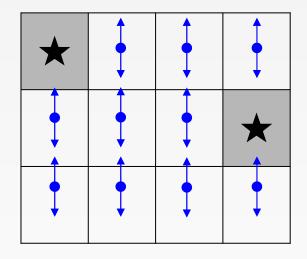


Set a negative "reward" for each transition (e.g. r = -1)

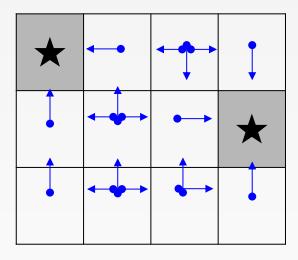
Objective: reach one of terminal states (greyed out) in least number of actions



A SIMPLE MDP: GRID WORLD



Random Policy



Optimal Policy



THE OPTIMAL POLICY π*

We want to find optimal policy π^* that maximizes the sum of rewards.



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How do we handle the randomness (initial state, transition probability...)? Maximize the **expected sum of rewards!**



THE OPTIMAL POLICY π^*

We want to find optimal policy π^* that maximizes the sum of rewards.

How do we handle the randomness (initial state, transition probability...)? Maximize the **expected sum of rewards!**

Formally:
$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$$
 with $s_0 \sim p(s_0), a_t \sim \pi(\cdot|s_t), s_{t+1} \sim p(\cdot|s_t, a_t)$



DEFINITIONS: VALUE FUNCTION AND Q-VALUE FUNCTION

Following a policy produces sample trajectories (or paths) s_0 , a_0 , r_0 , s_1 , a_1 , r_1 , ...



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How good is a state?

The **value function** at state s, is the expected cumulative reward from following the policy from state s: $\Gamma \qquad \qquad \rceil$

$$V^{\pi}(s) = \mathbb{E}\left[\left. \sum_{t \geq 0} \gamma^t r_t | s_0 = s, \pi
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How good is a state-action pair?

The **Q-value function** at state s and action a, is the expected cumulative reward from taking action a in state s and then following the policy:

$$Q^{\pi}(s,a) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi
ight]$$



BELLMAN EQUATION

The optimal Q-value function Q* is the maximum expected cumulative reward achievable from a given (state, action) pair:

$$Q^*(s,a) = \max_{\pi} \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi
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Q* satisfies the following **Bellman equation**:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Intuition: if the optimal state-action values for the next time-step $Q^*(s',a')$ are known, then the optimal strategy is to take the action that maximizes the expected value of

$$r + \gamma Q^*(s', a')$$



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$$r + \gamma Q^*(s', a')$$

The optimal policy π^* corresponds to taking the best action in any state as specified by Q*



Value iteration algorithm: Use Bellman equation as an iterative update

$$Q_{i+1}(s, a) = \mathbb{E}\left[r + \gamma \max_{a'} Q_i(s', a') | s, a\right]$$

Q_i will converge to Q* as i -> infinity

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Not scalable. Must compute Q(s,a) for every state-action pair. If state is e.g. current game state pixels, computationally infeasible to compute for entire state space!



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Solution: use a function approximator to estimate Q(s,a).



SOLVING FOR THE OPTIMAL POLICY

Value iteration algorithm: Use Bellman equation as an iterative update

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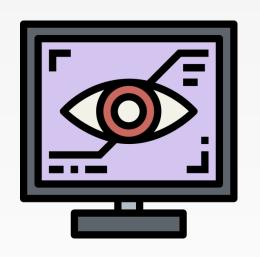
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Solution: use a function approximator to estimate Q(s,a). E.g. a neural network!





Q-LEARNING



Q-learning: Use a function approximator to estimate the action-value function

$$Q(s, a; \theta) \approx Q^*(s, a)$$



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If the function approximator is a deep neural network => **deep q-learning**!



Q-learning: Use a function approximator to estimate the action-value function

$$Q(s,a;\theta) \approx Q^*(s,a)$$
 function parameters (weights)

If the function approximator is a deep neural network => **deep q-learning**!



Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$



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$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function:
$$L_i(\theta_i) = \mathbb{E}_{s,a \sim \rho(\cdot)} \left[(y_i - Q(s,a;\theta_i))^2 \right]$$

where
$$y_i = \mathbb{E}_{s' \sim \mathcal{E}}\left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) | s, a
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Backward Pass

Gradient update (with respect to Q-function parameters θ):

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i)) \nabla_{\theta_i} Q(s, a; \theta_i) \right]$$



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CASE STUDY: PLAYING ATARI GAMES



Objective: Complete the game with the highest score

State: Raw pixel inputs of the game state

Action: Game controls e.g. Left, Right, Up, Down

Reward: Score increase/decrease at each time step



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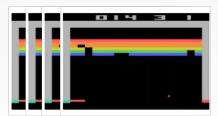
Q(s,a; heta) neural network with weights heta

FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4



Current state s_t: 84x84x4 stack of last 4 frames



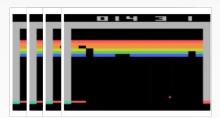
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Input: state s_t

Current state s_t: 84x84x4 stack of last 4 frames



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FC layer

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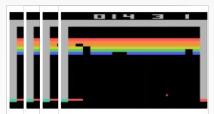
FC-4 (Q-values)

FC-256

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Last FC layer has 4-d output (if 4 actions), corresponding to Q(s_t, a₁), Q(s_t, a₂), Q(s_t, a₃), Q(s_t, a₄)



Current state s_t: 84x84x4 stack of last 4 frames



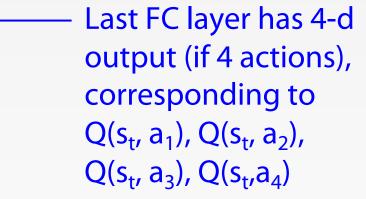
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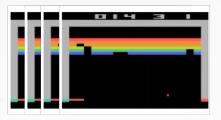
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Number of actions between 4-18 depending on Atari game

Current state s_t: 84x84x4 stack of last 4 frames



Q(s,a; heta) neural network with weights heta

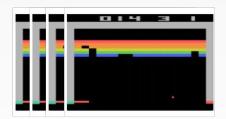
A single feedforward pass to compute Q-values for all actions from the current state => efficient!

FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4



Last FC layer has 4-d output (if 4 actions), corresponding to $Q(s_t, a_1)$, $Q(s_t, a_2)$, $Q(s_t, a_3)$, $Q(s_t, a_4)$

Number of actions between 4-18 depending on Atari game

Current state s_t: 84x84x4 stack of last 4 frames



NING THE Q-NETWORK: LOSS FUNCTION (FROM

Remember: want to find a Q-function that satisfies the Bellman Equation:

$$Q^*(s, a) = \mathbb{E}_{s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q^*(s', a') | s, a \right]$$

Forward Pass

Loss function:
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Backward Pass

Gradient update (with respect to Q-function parameters
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):

Iteratively try to make the Q-value close to the target value
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 it should have, if Q-function corresponds to optimal Q* (and optimal policy π^*)

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbb{E}_{s, a \sim \rho(\cdot); s' \sim \mathcal{E}} \left[r + \gamma \max_{a'} Q(s', a'; \theta_{i-1}) - Q(s, a; \theta_i)) \nabla_{\theta_i} Q(s, a; \theta_i) \right]$$



TRAINING THE Q-NETWORK: EXPERIENCE REPLAY

Learning from batches of consecutive samples is problematic:

- Samples are correlated => inefficient learning
- Current Q-network parameters determines next training samples (e.g. if maximizing action is to move left, training samples will be dominated by samples from left-hand size)
 => can lead to bad feedback loops



TRAINING THE Q-NETWORK: EXPERIENCE REPLAY

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Address these problems using experience replay

- Continually update a **replay memory** table of transitions (s_t, a_t, r_t, s_{t+1}) as game (experience) episodes are played
- Train Q-network on random minibatches of transitions from the replay memory, instead
 of consecutive samples



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- Train Q-network on random minibatches of transitions from the replay memory, instead
 of consecutive samples

Each transition can also contribute to multiple weight updates => greater data efficiency



Algorithm 1 Deep Q-learning with Experience Replay

```
Initialize replay memory \mathcal{D} to capacity N
Initialize action-value function Q with random weights
for episode = 1, M do
    Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
    for t = 1, T do
         With probability \epsilon select a random action a_t
         otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
         Execute action a_t in emulator and observe reward r_t and image x_{t+1}
         Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
         Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
         Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
         Set y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}
         Perform a gradient descent step on (y_j - Q(\phi_j, a_j; \theta))^2 according to equation 3
    end for
end for
```



Algorithm 1 Deep Q-learning with Experience Replay Initialize replay memory \mathcal{D} to capacity NInitialize replay memory, Initialize action-value function Q with random weights **Q-network** for episode = 1, M do Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$ for t = 1, T do With probability ϵ select a random action a_t otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$ Execute action a_t in emulator and observe reward r_t and image x_{t+1} Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$ Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D} Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D} Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$ Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3 end for end for



Algorithm 1 Deep Q-learning with Experience Replay Initialize replay memory \mathcal{D} to capacity NInitialize action-value function Q with random weights Play M episodes (full games) for episode = 1, M do Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$ for t = 1, T do With probability ϵ select a random action a_t otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$ Execute action a_t in emulator and observe reward r_t and image x_{t+1} Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$ Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D} Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D} Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$ Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3 end for end for



Algorithm 1 Deep Q-learning with Experience Replay

```
Initialize replay memory \mathcal{D} to capacity N
Initialize action-value function Q with random weights
for episode = 1, M do
     Initialise sequence s_1 = \{x_1\} and preprocessed sequenced \phi_1 = \phi(s_1)
    for t = 1, T do
          With probability \epsilon select a random action a_t
          otherwise select a_t = \max_a Q^*(\phi(s_t), a; \theta)
          Execute action a_t in emulator and observe reward r_t and image x_{t+1}
          Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
          Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
         Sample random minibatch of transitions (\phi_j, a_j, r_j, \phi_{j+1}) from \mathcal{D}
         Set y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}
```

Perform a gradient descent step on $(y_i - Q(\phi_i, a_i; \theta))^2$ according to equation 3

Initialize state (starting game screen pixels) at the beginning of each episode

end for

end for

Algorithm 1 Deep Q-learning with Experience Replay

```
Initialize replay memory \mathcal{D} to capacity N
Initialize action-value function Q with random weights

for episode =1,M do

Initialise sequence s_1=\{x_1\} and preprocessed sequenced \phi_1=\phi(s_1)

for t=1,T do
```

With probability ϵ select a random action a_t otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

Execute action a_t in emulator and observe reward r_t and image x_{t+1}

Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

Set
$$y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$$

Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

For each timestep t of the game



policy

PUTTING IT TOGETHER: DEEP Q-LEARNING WITH EXPERIENCE REPLAY

Algorithm 1 Deep Q-learning with Experience Replay Initialize replay memory \mathcal{D} to capacity NInitialize action-value function Q with random weights for episode = 1, M do Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$ for t = 1, T do With probability ϵ select a random action a_t With small otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$ probability, select Execute action a_t in emulator and observe reward r_t and image x_{t+1} Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$ a random action Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D} (explore), Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D} Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$ otherwise select greedy action Perform a gradient descent step on $(y_i - Q(\phi_i, a_i; \theta))^2$ according to equation 3 from current



end for

end for

```
Algorithm 1 Deep Q-learning with Experience Replay
   Initialize replay memory \mathcal{D} to capacity N
   Initialize action-value function Q with random weights
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            Store transition (\phi_t, a_t, r_t, \phi_{t+1}) in \mathcal{D}
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            Perform a gradient descent step on (y_i - Q(\phi_i, a_i; \theta))^2 according to equation 3
       end for
   end for
```

Take the action (a_t) , and observe the reward r_t and next state s_{t+1}

Algorithm 1 Deep Q-learning with Experience Replay

```
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Initialize action-value function Q with random weights
for episode = 1, M do
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        Set s_{t+1} = s_t, a_t, x_{t+1} and preprocess \phi_{t+1} = \phi(s_{t+1})
```

Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D}

Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D}

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Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3

end for

end for

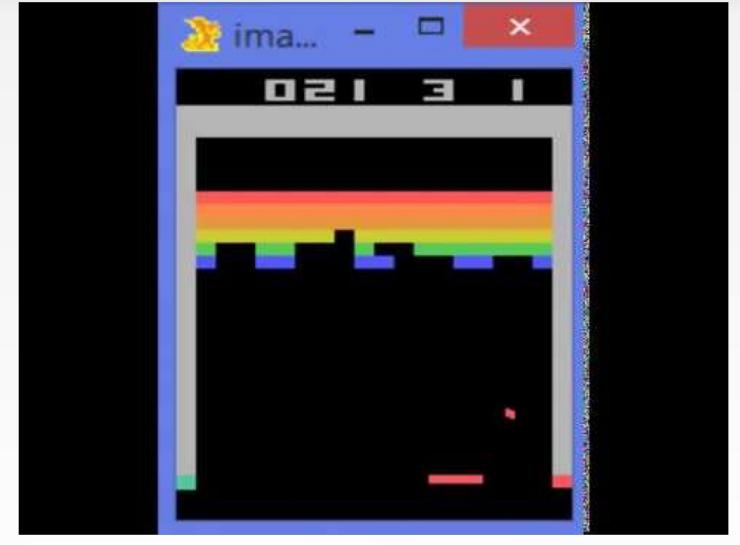
Store transition in replay memory



Algorithm 1 Deep Q-learning with Experience Replay Initialize replay memory \mathcal{D} to capacity NInitialize action-value function Q with random weights for episode = 1, M do Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$ for t = 1, T do With probability ϵ select a random action a_t otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$ Execute action a_t in emulator and observe reward r_t and image x_{t+1} Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$ Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in \mathcal{D} Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from \mathcal{D} Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$ Perform a gradient descent step on $(y_i - Q(\phi_i, a_i; \theta))^2$ according to equation 3 end for end for

Experience Replay:
Sample a random
minibatch of
transitions from replay
memory and perform a
gradient descent step





https://www.youtube.com/watch?v=V1eYniJ0Rnk

Video by Károly Zsolnai-Fehér. Reproduced with permission.



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What is a problem with Q-learning? The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair



What is a problem with Q-learning? The Q-function can be very complicated!

Example: a robot grasping an object has a very high-dimensional state => hard to learn exact value of every (state, action) pair

But the policy can be much simpler: just close your hand Can we learn a policy directly, e.g. finding the best policy from a collection of policies?







Formally, let's define a class of parameterized policies: $\Pi = \{\pi_{\theta}, \theta \in \mathbb{R}^m\}$

For each policy, define its value:

$$J(heta) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | \pi_ heta
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We want to find the optimal policy $\qquad \theta^* = rg \max_{ heta} J(heta)$

$$\theta^* = \arg\max_{\theta} J(\theta)$$

How can we do this?

Gradient ascent on policy parameters!



Mathematically, we can write:

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)} [r(\tau)]$$
$$= \int_{\tau} r(\tau) p(\tau;\theta) d\tau$$

Where $ext{r}(au)$ is the reward of a trajectory $au=(s_0,a_0,r_0,s_1,\ldots)$

Expected reward: $J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}\left[r(\tau)\right]$ $= \int_{\tau} r(\tau)p(\tau;\theta)\mathrm{d}\tau$

Expected reward: $J(\theta) = \mathbb{E}_{ au \sim p(au; heta)} \left[r(au)
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Now let's differentiate this: $\nabla_{ heta}J(heta)=\int_{ au}r(au)\nabla_{ heta}p(au; heta)\mathrm{d} au$



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Intractable! Gradient of an expectation is problematic when p depends on θ



Expected reward: $J(\theta) = \mathbb{E}_{ au \sim p(au; heta)} \left[r(au)
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Now let's differentiate this: $abla_{ heta}J(heta)$

$$abla_{ heta}J(heta) = \int_{ au} r(au)
abla_{ heta} p(au; heta) \mathrm{d} au$$

Intractable! Gradient of an expectation is problematic when p depends on θ

However, we can use a nice trick:

$$\nabla_{\theta} p(\tau;\theta) = p(\tau;\theta) \frac{\nabla_{\theta} p(\tau;\theta)}{p(\tau;\theta)} = p(\tau;\theta) \nabla_{\theta} \log p(\tau;\theta)$$



Expected reward:

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$$= \int_{\tau} r(\tau) p(\tau;\theta) d\tau$$

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$$\nabla_{\theta} J(\theta) = \int_{\tau} r(\tau) \nabla_{\theta} p(\tau; \theta) d\tau$$

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$$\nabla_{\theta} J(\theta) = \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau$$
$$= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)]$$

Can estimate with Monte Carlo sampling



Can we compute those quantities without knowing the transition probabilities?

We have:
$$p(\tau; \theta) = \prod_{t>0} p(s_{t+1}|s_t, a_t) \pi_{\theta}(a_t|s_t)$$



Can we compute those quantities without knowing the transition probabilities?

We have:
$$p(\tau; \theta) = \prod p(s_{t+1}|s_t, a_t)\pi_{\theta}(a_t|s_t)$$

Thus:
$$\log p(\tau; \theta) = \sum_{t \ge 0}^{t \ge 0} \log p(s_{t+1}|s_t, a_t) + \log \pi_{\theta}(a_t|s_t)$$



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And when differentiating:
$$abla_{ heta} \log p(au; heta) = \sum_{t \geq 0}
abla_{ heta} \log \pi_{ heta}(a_t | s_t)$$

Doesn't depend on transition probabilities!



$$\nabla_{\theta} J(\theta) = \int_{\tau} (r(\tau) \nabla_{\theta} \log p(\tau; \theta)) p(\tau; \theta) d\tau$$
$$= \mathbb{E}_{\tau \sim p(\tau; \theta)} [r(\tau) \nabla_{\theta} \log p(\tau; \theta)]$$

Can we compute those quantities without knowing the transition probabilities?

We have:
$$p(\tau;\theta) = \prod_{t=0}^{\infty} p(s_{t+1}|s_t, a_t) \pi_{\theta}(a_t|s_t)$$

Thus:
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And when differentiating:
$$\nabla_{\theta} \log p(\tau; \theta) = \sum_{t \geq 0} \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Doesn't depend on transition probabilities!

Therefore when sampling a trajectory τ , we can estimate $J(\theta)$ with

$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$



INTUITION

Gradient estimator: $\nabla_{\theta} J(\theta) pprox \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

Interpretation:

- If $r(\tau)$ is high, push up the probabilities of the actions seen
- If $r(\tau)$ is low, push down the probabilities of the actions seen



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Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!



INTUITION

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Might seem simplistic to say that if a trajectory is good then all its actions were good. But in expectation, it averages out!

However, this also suffers from high variance because credit assignment is really hard. Can we help the estimator?



VARIANCE REDUCTION

Gradient estimator: $\nabla_{\theta} J(\theta) pprox \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$



VARIANCE REDUCTION

Gradient estimator: $\nabla_{\theta} J(\theta) pprox \sum_{t \geq 0} r(\tau) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$

First idea: Push up probabilities of an action seen, only by the cumulative future reward from that state

$$abla_{\theta} J(\theta) pprox \sum_{t \geq 0} \left(\sum_{t' \geq t} r_{t'} \right)
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VARIANCE REDUCTION

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$$\nabla_{\theta} J(\theta) pprox \sum_{t \geq 0} \left(\sum_{t' \geq t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

Second idea: Use discount factor γ to ignore delayed effects

$$\nabla_{\theta} J(\theta) pprox \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$



VARIANCE REDUCTION: BASELINE

Problem: The raw value of a trajectory isn't necessarily meaningful. For example, if rewards are all positive, you keep pushing up probabilities of actions.

What is important then? Whether a reward is better or worse than what you expect to get

Idea: Introduce a baseline function dependent on the state.

Concretely, estimator is now:

$$\nabla_{\theta} J(\theta) pprox \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$



$$\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} \left(\sum_{t' \geq t} \gamma^{t'-t} r_{t'} - b(s_t) \right) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$$

A simple baseline: constant moving average of rewards experienced so far from all trajectories



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A simple baseline: constant moving average of rewards experienced so far from all trajectories

Variance reduction techniques seen so far are typically used in "Vanilla REINFORCE"



A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?



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A: Q-function and value function!



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Intuitively, we are happy with an action a_t in a state s_t if $Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$ is large. On the contrary, we are unhappy with an action if it's small.



A better baseline: Want to push up the probability of an action from a state, if this action was better than the **expected value of what we should get from that state**.

Q: What does this remind you of?

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Intuitively, we are happy with an action a_t in a state s_t if $Q^{\pi}(s_t, a_t) - V^{\pi}(s_t)$ is large. On the contrary, we are unhappy with an action if it's small.

Using this, we get the estimator: $\nabla_{\theta} J(\theta) \approx \sum_{t \geq 0} (Q^{\pi_{\theta}}(s_t, a_t) - V^{\pi_{\theta}}(s_t)) \nabla_{\theta} \log \pi_{\theta}(a_t | s_t)$



ACTOR-CRITIC ALGORITHM

Problem: we don't know Q and V. Can we learn them?

Yes, using Q-learning! We can combine Policy Gradients and Q-learning by training both an **actor** (the policy) and a **critic** (the Q-function).

- The actor decides which action to take, and the critic tells the actor how good its action was and how it should adjust
- Also alleviates the task of the critic as it only has to learn the values of (state, action) pairs generated by the policy
- Can also incorporate Q-learning tricks e.g. experience replay
- Remark: we can define by the advantage function how much an action was better than expected $4\pi(s) = 0$

$$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$$



ACTOR-CRITIC ALGORITHM

Initialize policy parameters θ , critic parameters ϕ For iteration=1, 2 ... do Sample m trajectories under the current policy $\Delta \theta \leftarrow 0$ **For** i=1, ..., m **do** For t=1, ..., T do $A_t = \sum_{t' \ge t} \gamma^{t'-t} r_t^i - V_{\phi}(s_t^i)$ $\Delta \theta \leftarrow \Delta \theta + A_t \nabla_{\theta} \log(a_t^i | s_t^i)$ $\Delta \phi \leftarrow \sum_{i} \sum_{t} \nabla_{\phi} ||A_{t}^{i}||^{2}$ $\theta \leftarrow \alpha \Delta \theta$ $\phi \leftarrow \beta \Delta \phi$

End for



Objective: Image Classification

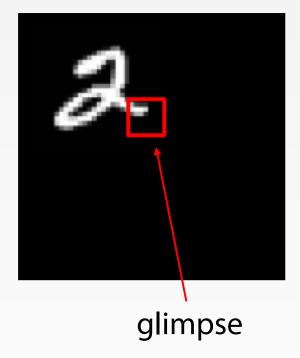
Take a sequence of "glimpses" selectively focusing on regions of the image, to predict class

- Inspiration from human perception and eye movements
- Saves computational resources => scalability
- Able to ignore clutter / irrelevant parts of image

State: Glimpses seen so far

Action: (x,y) coordinates (center of glimpse) of where to look next in image

Reward: 1 at the final timestep if image correctly classified, 0 otherwise





Objective: Image Classification

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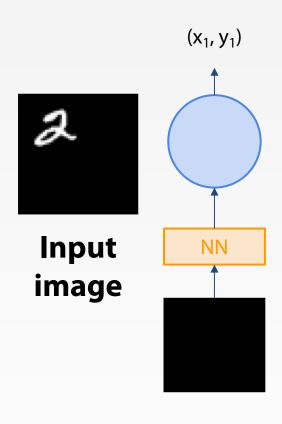
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glimpse

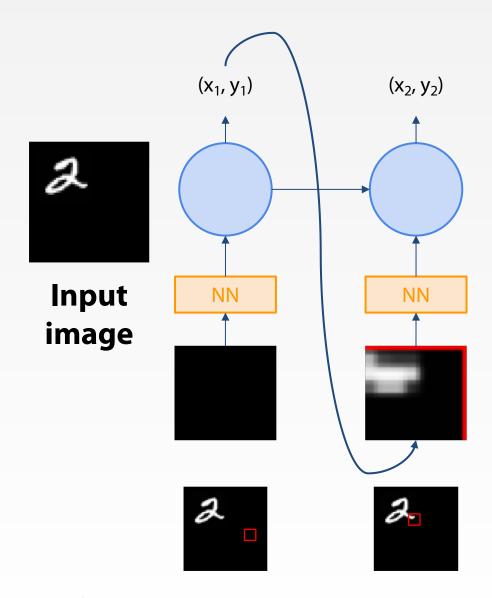
Glimpsing is a non-differentiable operation => learn policy for how to take glimpse actions using REINFORCE Given state of glimpses seen so far, use RNN to model the state and output next action







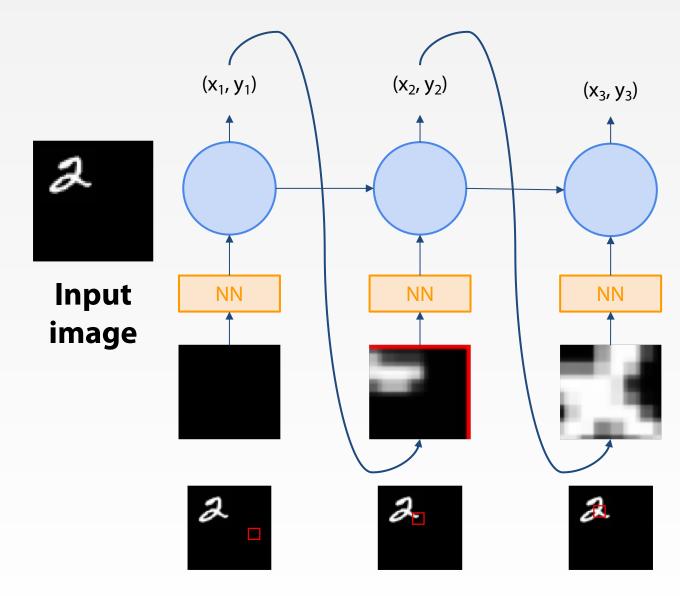




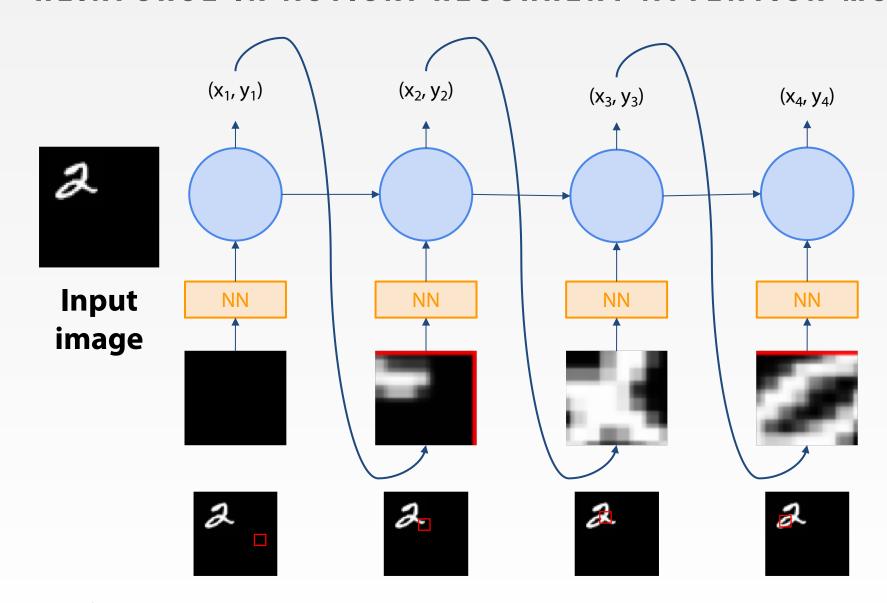
[Mnih et al. 2014]

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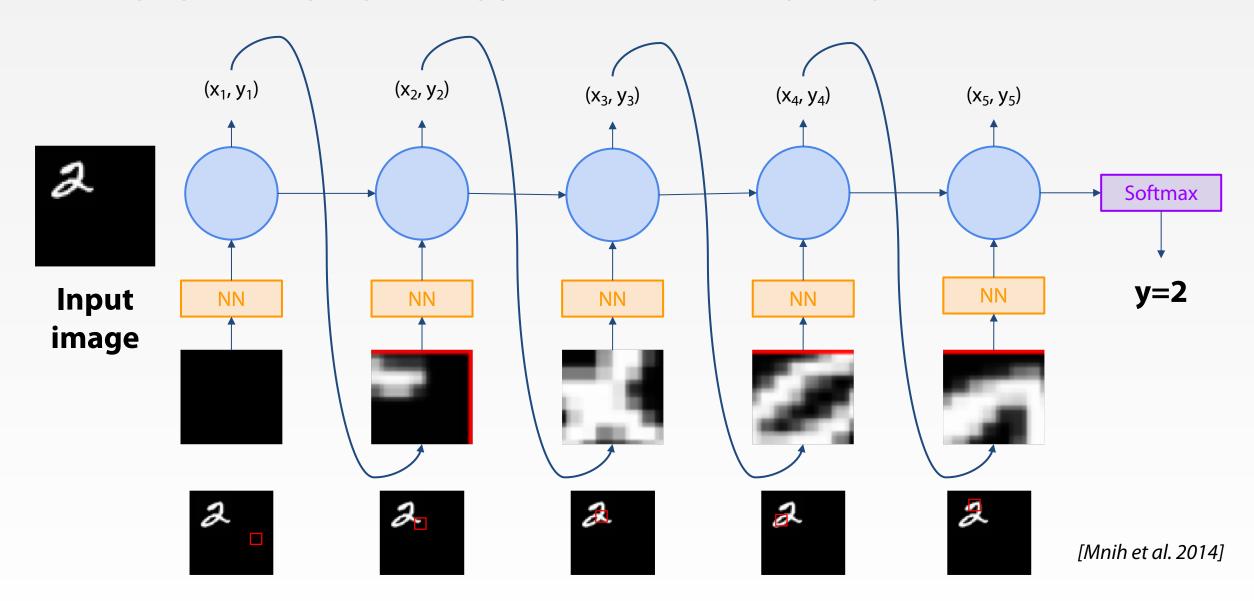




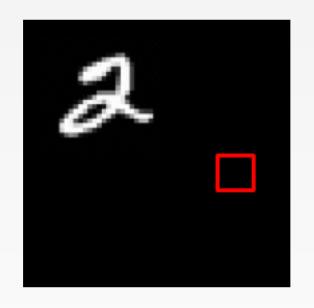
[Mnih et al. 2014]

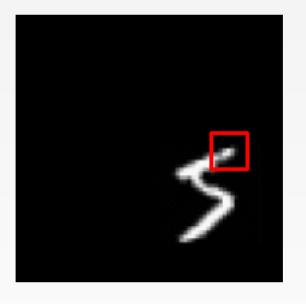


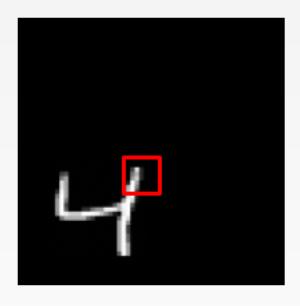
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Has also been used in many other tasks including fine-grained image recognition, image captioning, and visual question-answering!

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COMPETING AGAINST HUMANS IN GAME PLAY

AlphaGo [DeepMind, Nature 2016]:

- Required many engineering tricks
- Bootstrapped from human play
- Beat 18-time world champion Lee Sedol

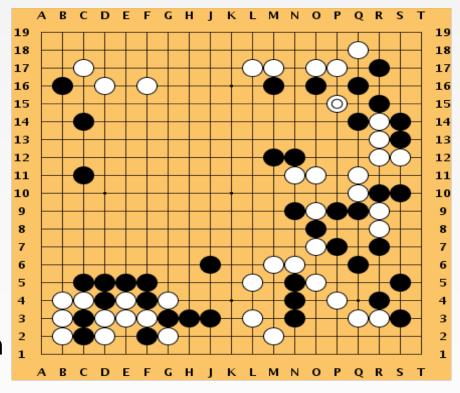
AlphaGo Zero [Nature 2017]:

- Simplified and elegant version of AlphaGo
- No longer bootstrapped from human play
- Beat (at the time) #1 world ranked Ke Jie

Alpha Zero: Dec. 2017

 Generalized to beat world champion programs on chess and shogi as well

Recent advances in more complex games, e.g. StarCraft (DeepMind) and Dota (OpenAI)





SUMMARY

Policy gradients:

- Very general but suffer from high variance so requires a lot of samples
- Challenge: sample-efficiency

Q-learning

- Does not always work but when it works, usually more sample-efficient
- Challenge: exploration

Guarantees:

- **Policy Gradients**: Converges to a local minima of $J(\theta)$, often good enough!
- Q-learning: Zero guarantees since you are approximating Bellman equation with a complicated function approximator



NEXT TIME: ADVERSARIAL TRAINING

