Problem 1, Quicksort, 20 points
Cormen-Leiserson-Rivest, Problem 8-4 concerning the Stack depth of Quicksort (pages 169-170 in 1990 Edition). Parts (a) and (b) are worth 1 point each, and part (c) is worth 18 points.

Problem 2, Heapsort, 20 points
Cormen-Leiserson-Rivest, Exercise 7.2-4 concerning the implementation of Heapsify (page 144 in 1990 Edition).

Problem 3, Special Median, 20 points

Problem 4, Find the Sink, 20 points
Let $a_{ij}, 1 \leq i, j \leq n$ be the adjacency matrix of a directed graph. A sink in a directed graph is a vertex with incoming edges from all other vertices and without any outgoing edges. Thus, a sink is a vertex $i$ with $a_{ij}=0, \forall 1 \leq j \leq n$, and $a_{ji}=1, \forall 1 \leq j \leq n, j \neq i$.

a. (1 point) Argue that a directed graph can have at most one sink.

b. (19 points) Give an algorithm that decides if a directed graph given by its adjacency matrix has a sink and, if so, outputs $a_{ij}, \forall 1 \leq j \leq n$, and $a_{ji}, \forall 1 \leq j \leq n, j \neq i$, where vertex $i$ is the sink. Your algorithm should use $O(n)$ probes to entries of the adjacency matrix (ie, while the adjacency matrix has $n^2$ entries, the algorithm reads only $O(n)$ of them).

Problem 5, Generate all Permutations, 20 points
Write an algorithm to generate all permutations of the integers 1 to $n$. Your algorithm should use work and output space polynomial in $n$; for example, on input 3, an output array may successively take values 321, 312, 231, 132, 213 and 123. Hint: The set of permutations of the integers 1 to $n$ can be obtained from the set of permutations of the integers 1 to $n-1$ by inserting $n$ in each possible position of the permutation.