This homework has a total of 3 problems on 1 page. Solutions should be submitted to Canvas before 12:00pm on Thursday Feb 13.

The problem set is marked out of 25, you can earn up to $1 + 8 + 7 + 10 = 26$ points.

Please refer to the homework guidelines for formatting as well as policy on late days. If you choose not to submit a typed write-up, please write neat and legibly.

Collaboration is allowed/encouraged on problems, however each student must independently complete their own write-up, and list all collaborators. No credit will be given to solutions obtained verbatim from the Internet or other sources.

0. (1 point) Submit your homework in accordance to the guidelines at
https://www.cc.gatech.edu/~rpeng/CS4510_S20/HomeworkGuidelines.pdf.

1. (8 points) Automata and Regular Expressions

   (a) (4 points) Convert the NFA below to a regular expression

   
   ![NFA Diagram]

   (b) (4 points) Give an NFA that accepts the language described by the regular expression

   $$(((a \circ b) \cup (b \circ a))^*.$$

2. (7 points) Common Pumping Lemma Misunderstandings

   (a) (2 points) You are a CS4510 TA, and you are grading the following question:

   “Is the language

   $$L = \{w \mid w \text{ is a binary string with at least as many 1’s as 0’s}\}$$

   regular? Prove your answer.”

   1
Bob, a student in the class, has submitted the following answer:

I will prove that $L$ is not regular, using the pumping lemma.
   i. Let $p \in \mathbb{Z}^+$.
   ii. Let $w = 01^p$, so that $w \in L$ and $|w| \geq p$.
   iii. Let $w = xyz$, with $x = \epsilon$, $y = 0$, and $z = 1^p$, so that $|xy| \leq p$ and $|y| > 0$.
   iv. Then $xy^{p+1}z$ is not in $L$.

Identify the single incorrect line in Bob’s proof and explain what he did wrong.

(NOTE: do not list small mistakes, typos, etc.; explain the major logical error which invalidates Bob’s proof.)

(b) (2 points) Now you are grading the following problem:
   “Let $L$ be any finite, nonempty language of binary strings. Is $L$ regular?”

Katharine has submitted the following answer:

I will prove that $L$ is not regular, using the pumping lemma.
   i. Since $L$ is finite, it has a longest string. Let $p$ be the length of the longest string in $L$.
   ii. Let $w \in L$, such that $|w| \geq p$.
   iii. Let $w = xyz$, so that $|xy| \leq p$ and $|y| > 0$.
   iv. Then $xy^2z$ is not in $L$, since it has length longer than $p$, but all the strings in $L$ have length at most $p$.

Identify the single incorrect line in Katharine’s proof, and explain what she did wrong.

(NOTE: Do not just say, “$L$ is actually regular”; explain the major logical error which invalidates Katharine’s proof.)

(c) (3 points) Consider “Pumping Lemma 2”, given below:

If
   i. for all positive integers $p$,
   ii. there exists a word $w \in L$ with $|w| > p$ such that
   iii. there exists a split of $w = xyz$ with $|xy| \leq p$ and $|y| > 0$ such that
   iv. for some $i$, $xy^iz \notin L$.

then $L$ is not regular.

Note that Pumping Lemma 2 is not true! However, for this problem, we are going to pretend that it is.

Give a proof, using Pumping Lemma 2, that the language

$$L = \{ w | w \text{ is a binary string with an even number of ones} \}$$

is not regular.
3. (10 points) Pumping Lemma

(a) (5 points) Show that the language

\[ L = \{0^{2n}1^n \mid n \geq 0\} \]

is not regular using the pumping lemma.

(b) (5 points) Prove that

\[ L = \{a^ib^jc^k \mid i + j = k\} \]

is not regular using the pumping lemma.